Strategic Effects of Three-Part Tariffs under Oligopoly*

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Abstract

The distinct element of a three-part tariff, compared with linear pricing or a two-part tariff, is its quantity target within which the marginal price is zero. This quantity target instrument enriches the firm's strategy set in dictating the competition to a specific level, even in the absence of usual price discrimination motive. With general differentiated linear demands, the competitive effect of a three-part tariff in contrast to linear pricing depends on the degree of substitutability between products: competition is intensified when two products are more differentiated, yet softened when two products are more substitutable.

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1 Introduction

A three-part tariff (3PT) refers to a pricing scheme consisting of a fixed fee, a free allowance of units up to which the marginal price is zero, and a positive per-unit price for additional demand beyond that allowance. The 3PT and its variations are prevalent in many contexts, including both final-goods markets and intermediate-goods markets. At the end user level, 3PTs have become popular recently in information industries. Examples of 3PTs include the pricing structures for cellular phone plans, Internet access service, data center hosting, and "cloud computing".[1] As for the business level, 3PT contracts are commonly used by dominant firms and they have raised many antitrust concerns regarding their putative exclusionary effects.

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[1]For example, a typical cell phone plan from AT&T is $39.99/month for 450 mins, with $0.45/min for overtime calling. And in many European countries, Internet subscription pricing is a 3PT (See Lambrecht, Seim and Skiera, 2007). Other examples include data plans for iPhone and iPad 2. As for online data storage, RimuHosting charges $20/month for 30GB, with $1/GB for additional storage. In addition, RackSpace adopts a 3PT for "cloud computing" service—–$100/month for 50GB disk space, 500GB bandwidth and 3 million Web requests, with $0.50/GB additional disk space, $0.25/GB additional bandwidth and $0.03/1000 Web requests.

Although 3PTs have been widely used for quite some time, the economics and business literature concerning them has been rather sparse until recently. And within the limited literature, 3PTs are often either analogized to the canonical two-part tariff (2PT), or they are interpreted as a market segmenting tool when demand is uncertain. Oi (1971) was the first to mention IBM's 3PT contracts for its machine leasing in his classical "Disneyland Dilemma" article, and he interpreted it as a surplus extraction device for the monopoly in the same spirit as the 2PT. Bagh and Bhargava (2008) showed that a monopoly with a more ornate menu of 2PTs can be outperformed by a smaller menu of 3PTs. Both articles consider the 3PT from the perspective of a monopolist, and competition is assumed away. Lambrecht, Seim and Skiera (2007) developed a discrete/continuous choice model for empirical estimation using Internet usage data and showed that demand uncertainty plays a key role in consumer's behavior under 3PTs. Since their focus is on consumer's tariff choice and usage quantity decisions under 3PTs, they took specific 3PTs as given for all firms. Consequently, the optimality of 3PTs and the corresponding competitive effects when firms compete with each other are not considered in their article. Along the line of consumer's decision with demand uncertainty, Grubb (2009) incorporated consumers' overconfidence into a sequential screening model in which consumers have to sign a contract when they are uncertain about their eventual demand.[2] He used U.S. cellular phone data to support the superiority of the overconfidence assumption over the common priors assumption that both firm and consumers agree on the distribution of future demand when explaining a 3PT as a customer screening device. He restricted attention to monopoly and perfect competition cases, and thus strategic effects of 3PTs under imperfect competition are not issues in his model.

All these articles share a central theme—–explaining the 3PT as a price discrimination tool that helps firms segment heterogeneous consumers. More importantly, all the 3PTs discussed in the above articles are targeted to final consumers, who often buy solely from a single seller. Rather, in intermediate-goods markets, *the downstream firms typically carry products from more than one upstream suppliers*.[3] In fact, this is the level at which antitrust issues primarily arise due to limited competition and possible exclusion when purchasing from multiple sources is indeed possible. To the best of my knowledge, competitive effects of 3PTs in such a common agency context have, however, largely been unexplored to date.

So instead of looking at those final-good markets where consumers buy only from a single supplier, the motivation for this article comes from vertical contracts in intermediate-goods markets where multiple sourcing is common. In the landmark case, *U.S. v. Microsoft Corp.*, one of the alleged antitrust practices of Microsoft was with regard to its 3PT pricing structure for its CPU license. "*Under the CPU license, an OEM usually had to also commit to a minimum 'requirement'* (X)". "*(O)nce the contract is in place, the marginal price is 0 up to X units and f for additional units.*" (See Baseman, Warren-Boulton and Woroch, 1995).[4] What makes the 3PT appear anticompetitive is the large quantity threshold within which the marginal price is zero, a stipulation which may reduce the demand for the rivals' products to levels so low that the rivals are forced to exit the market, despite the fact that accepting the 3PT from the dominant firm does not necessarily or explicitly exclude the retailer's right to purchase from other suppliers.

In this article, we develop a game theoretical model to study the strategic reasons why a dom-

[2]For the sequential screening model, see Baron and Besanko (1984) and Courty and Li (2000).

[3]I thank an anonymous referee for bringing to my attention this essential disctinction differentiating my article from the existing literature mentioned above, that I had failed to recognize.

[4]Other antitrust cases involving 3PT contracts include *Barry Wright Corp. v. ITT Grinnell Corp.* and *Magnus Petroleum CO., Inc. v. Skelly Oil CO.*.

inant firm under oligopoly offers such a 3PT to downstream firms and the implications of this for antitrust concerns. We consider a sequential-move setting of two competing upstream manufacturers selling their substitute products to a single downstream retailer, with the dominant firm moving first and the rival firm as a follower in offering contracts to the retailer. To rule out price discrimination as a possible motive for the upstream firm a priori, we restrict attention to the case of a single buyer with complete information in which the demand and costs are common knowledge to all parties. Hence, it can be viewed as an extension of the classical Stackelberg model. Meanwhile, our model is a "sequential, delegated common agency" as defined by Bernheim and Whinston (1986) and Prat and Rustichini (1998). To assess the welfare implication, we compare 3PT equilibrium with the classical Stackelberg linear pricing (LP) equilibrium as well as 2PT equilibrium.

The main finding is that the manner in which a 3PT influences competition is strikingly different from that of LP or a 2PT: it could be *either* a “Top Dog Ploy” *or* a “Puppy Dog Ploy”.[5] We find that a 3PT is always a profitable tool over LP or a 2PT for the dominant firm to compete against its rival, in both cases of perfect substitutes and imperfect substitutes. We further perform comparative statics analysis to explore the effects of a 3PT on welfare using general differentiated linear demands. Compared with LP, a 2PT always intensifies competition in terms of offering higher total surplus and lowering the rival firm’s profit, but a 3PT may, contrastingly, lower the total surplus and increase the rival firm’s profit. Interestingly, the competitive effects of a 3PT in contrast to LP depend on the degree of substitutability between products: competition is *intensified (“Top Dog”)* when two products are more differentiated, but *softened (“Puppy Dog”)* when two products are more substitutable. Remarkably, although a 3PT is more ornate than a 2PT, it is always the case that a 3PT lessens competition over a 2PT. Furthermore, we have shown that when taking into account the sunk cost from the follower and the associated entry decision, the 3PT will only enlarge the set of welfare-reducing exclusion.

The central idea of the article is that the 3PT can be a credible commitment tool for the dominant firm to set the tone for competition and induce the rival firm’s optimal responses toward the interests of the dominant firm through a judiciously designed 3PT. Note that compared with LP or a 2PT, the distinct element from the 3PT is its quantity target. By breaking its pricing scheme into two blocks, with zero marginal price for the first block ranging from zero unit to the quantity target, and with a positive marginal price afterwards, the leader becomes free from the “applying-to-all-units” pricing feature of a single per-unit price under either LP or a 2PT. So this quantity target instrument enriches the leader’s strategy set in dictating the competition to a specific level.

What makes the 3PT interesting is that the single instrument—quantity target can actually play two opposite roles. When two products are more homogeneous, the quantity target will be a handcuff for manufacturer A to tie his own hand in order to avoid fierce competition. When two products are more differentiated, the quantity target will behave more as a surplus extraction device, in the same spirit as the quantity forcing contract, and thus intensify competition.

The remainder of the article is organized as follows. In Section 2, we set up the model and describe the game. Section 3 presents and analyzes both cases of perfect substitutes and imperfect substitutes. In Section 4, we perform comparative statics and discuss properties of the equilibriums under all three pricing regimes, as well as their implications on competition. Section 5 presents some extensions. First, it addresses the possible exclusion of the 3PT by introducing a sunk cost for

[5]“Top Dog Ploy” refers to a strategy of being tough and aggressive to compete fiecely against the rival, and “Puppy Dog Ploy” is a metaphor for being small and friendly to induce accommodation from the rival. For discussions on these strategies, see Fudenberg and Tirole (1984).

the rival firm. Second, it discusses the case when the rival firm can use a 3PT. The article closes in Section 6 with some concluding remarks. All proofs are relegated to the Appendices.

2 The Model

In this section, we set up a model to analyze a 3PT in a competitive environment, and we describe how the game proceeds.

Generally, a 3PT consists of a triple (T_o, Q_o, w), where T_o is the fixed fee for the right to stock supplier's product, Q_o is the quantity threshold within which it is free of charge, and w is the per-unit price for the quantities exceeding the threshold Q_o. Specifically, the 3PT total payment schedule is

$$T(q) = \begin{cases} T_o & \text{if } q < Q_o \\ T_o + w(q - Q_o) & \text{if } q \geq Q_o \end{cases} .$$

And it is illustrated in Figure 1.

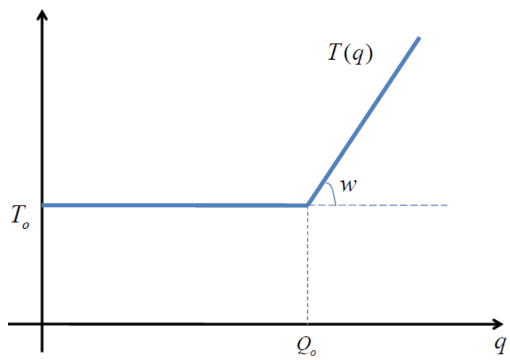


Figure 1: 3PT Total Payment Schedule

The model consists of two classes of agents. First, two manufacturers A and B are located in the upstream market and produce substitute products with the same marginal cost c. In this set-up, the two products manufactured by A and B are allowed to be general substitutes, including both cases of homogeneous products and differentiated products. Second, there are a large number of retailers, each of whom is a local monopoly in selling to final consumers.[6] We assume complete information about demands in every retailing market here, and two manufacturers make customized offers to each local monopoly retailer. Hence, it is without loss of generality to consider a representative retailer R in the downstream with a retailing revenue function denoted as $R(q_A, q_B)$. For simplicity, we assume that retailers have no cost other than the wholesale prices charged by the manufacturers.

As our objective here is to see if a 3PT can have any strategic effects purely coming from upstream competition, we want to rule out any other motives as best as we can. The local monopoly retailer assumption helps us abstract away from strategic interactions resulting from downstream competition. In addition, the complete information assumption in the model prevents price discrimination from being a plausible explanation. As will be illustrated soon, even in this simple framework, a 3PT has some bite on competition in a strikingly different way from LP or a 2PT.

[6]This corresponds to the market structure where there are a large number of buyers while only few sellers, as a typical case in which antitrust concerns on contracts offered by those dominant upstream firms arise. For example, in our motivating example *U.S. v. Microsoft Corp.*, the downstream buyers don't have many alternative suppliers of operating systems. Mathewson and Winter (1987) made such an assumption, too.

The game is a sequential-move game involving three stages. At date 1, manufacturer A offers a contract to the retailer. The contract is, in general, a 3PT contract (T_o, Q_o, w_A).[7] After observing the contract offer from manufacturer A, at date 2, manufacturer B sets its per-unit wholesale price w_B for the retailer. Here we confine attention to LP from B in order to capture the fact that the small firm in reality usually cannot match the contract as complicated as offered by a dominant firm. In *U.S. v. Microsoft Corp.*, only LP is offered by the rival firm. This contract space restriction is critical for our results on the 3PT. The cases when this assumption is relaxed are discussed in subsection 5.2. At date 3, the retailer R decides what to buy. It is worth noting that the retailer here can purchase from both manufacturers. Remember that there are many local monopoly retailers here, but only two manufacturers. Therefore, it is reasonable to assume that those retailers have no monopsony power. For completeness, we assume that in the event of a tie when the retailer is indifferent between buying from A and B, he will buy from B only. This tie-breaking rule is purely to avoid the need to consider a follower charge a price arbitrarily close to, but below the leader's price. The game's timeline is described in Figure 2.

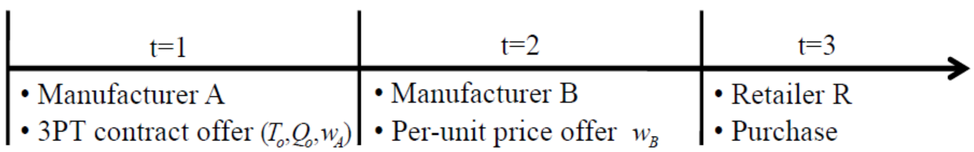


Figure 2: Game's Timeline

For the timing of the game, in practice, the 3PT becomes an antitrust concern only when the firm adopting it enjoys a dominant position in the market. When there is a dominant firm, it is the dominant firm usually moves first, and the number of moves is small.[8] In theory, as shown in Corollary 1, both manufacturers earn higher profit when they move sequentially than when they move simultaneously. This implies that if we allow the choice of competing manufacturers to move simultaneously or sequentially to be endogenously determined, then they both will adopt the sequential move. Further, the endogenous price leadership literature shows that the dominant firm will emerge as the price leader.[9] Here we model manufacturer A as the dominant firm due to which it moves first and offers a more complicated contract than the follower B does.

Moreover, the assumption that the retailer only makes its purchase decision until two competing offers are on the table is to capture the contestable condition for the favor of retailers. It is worth noting that the nature of sequential-move game here is different from that first introduced by Aghion and Bolton (1987) and then extended by Marx and Shaffer (2004). In particular, they consider a three-stage game in which the buyer negotiates a contract with each seller sequentially and then makes its purchase decision. In their settings, both contracts are binding once in place. As one can

[7]Note that when $Q_o = 0$, the 3PT is reduced to the classical 2PT. Furthermore, if we have $T_o = Q_o = 0$, then it becomes a uniform linear price schedule.

[8]"*Price leadership probably works best and arises most frequently in industries in which a single firm is outstanding by virtue of large size or recognized high quality of management*" (See Oxenfeldt, 1951, pp. 296). For instance, historically, GM is usually acknowledged as a price leader and Chrysler is widely recognized as a follower. Moreover, the leader doesn't change the price that often once it is settled. "*The latest Chrysler price boosts followed increases by GM, the industry's traditional price leader, and American Motors Corp. No. 2 Ford Motor Co. has said it doesn't expect any further price increases in the remaining three months of the 1980-model year*" (See "Chrysler, Following Other Auto Firms, Raises Prices 2.2%" on Wall Street Journal, Jul. 10th, 1980).

[9]For the endogenous price leadership literature, see Deneckere and Kovenock (1992), van Damme and Hurkens(2004).

expect, shifting rent from the second seller to the first seller and the buyer is possible there, since the specified transfer from the buyer to the first seller according to the binding contract in stage 1 becomes credible in stage 2's negotiation with the second seller. On the contrary, the order of moves in our setting automatically excludes the possibility of rent shifting between the buyer with any seller, because neither contract is binding for the buyer until the buyer purchases from it in the last stage. Additionally, the equilibrium strategies are renegotiation-proof by nature of the timing, since the retailer doesn't commit to any contract before both manufacturers make offers.[10] The nice aspect of this article is that even in this substantially competitive environment at upstream level, the 3PT still has interesting strategic effects.

In this setting, we determine subgame perfect equilibrium outcomes and analyze the equilibrium as the degree of product differentiation varies.

3 Equilibrium Analysis

3.1 The Case of Perfect Substitutes

We begin with the case of perfect substitutes. It illustrates the basic idea of how a 3PT affects competition in a simple setting, and thus helps us understand the distinctions between a 3PT and LP or a 2PT.

When two products are homogeneous, the retailing revenue function depends only on the sum of the quantities. That is, $R(q_A, q_B) = R(q_A + q_B)$. We denote the optimal quantity demanded by the retailer when he buys at per-unit price w as $q^m(w) \equiv \arg\max_{x\geq 0}[R(x) - w \cdot x]$, the corresponding monopoly profit earned by the sole supplier as $\pi^m(w) \equiv (w-c)\cdot q^m(w)$, and the optimal monopoly price as $w^m \equiv \arg\max_w \pi^m(w)$. We make the following assumptions.

Assumption 1 (Monotonicity and Concavity of $R(q)$) *$R(q)$ is $\mathbb{C}^2$ on $\mathbb{R}_+$. $R'(q) > 0, \forall q \in [0, \widehat{q})$; $R''(q) < 0, \forall q \in \mathbb{R}_+$. Here $\widehat{q}$ is the finite satiation quantity. That is, $\widehat{q} \equiv \arg\max_{q\geq 0} R(q)$.*

Assumption 1 implies that $q^m(w)$ is a well-defined continuously differentiable function with $q^{m\prime}(w) < 0, \forall w \in [0, R'(0))$. Moreover, it ensures that $\pi^m(w)$ is $\mathbb{C}^1$, $\forall w \in [0, R'(0))$.

Assumption 2 (Efficiency Requires Positive Sales) $0 \leq c < R'(0)$.

Assumptions 1 and 2 guarantee that $q^m(c) \in (0, \infty)$.

Assumption 3 (Concavity of Profit Function) *$\pi^{m\prime\prime}(w)$ exists and $\pi^{m\prime\prime}(w) \leq 0, \forall w \geq c$.*[11]

To ensure the existence and uniqueness of the 3PT equilibrium, we make the following technical assumption.

Assumption 4 (Single-Peakedness) *Assume $h(w) \equiv (w - c)\pi^{m\prime}(w)$ is single-peaked in $[c, w^m]$.*

[10] Semenov and Wright (2011) showed that with downstream competition, a generalized all-units discount can be renegotiation-proof in deterring entry, through a predatory pricing commitment. Our renegotiation-proof result doesn't rely on downstream competition, and the adoption of 3PT here is not distorted by the attempt of predation.

[11] $\pi^{m\prime\prime}(w) = 2q^{m\prime}(w) + (w - c) \cdot q^{m\prime\prime}(w)$. So a sufficient condition for the concavity of the profit function is that the demand function is weakly concave.

This assumption says $h(w)$ has a unique peak in the range between cost and monopoly price. This single-peakedness assumption is rather mild. As can be verified, both general linear demands, general constant elasticity demands and general exponential demands satisfy this assumption.

As a benchmark, we first look at the situation in which the leading firm can only offer a linear price or a 2PT, where the Bertrand paradox prevails. After that, we will see how the dominant firm can gain strictly positive profit through a 3PT in cases where neither LP nor a 2PT could work.

In the case of homogeneous products, no matter whether the leader adopts LP or a 2PT, a uniform per-unit price will be applied to all units purchased from him. Let $v(w) \equiv \max_{x \geq 0}[R(x) - w \cdot x]$ be the retailer's profit under a per-unit price w. Denote w_i as the per-unit price from manufacturer i $(i = A, B)$, and T and T_o as the fixed fees for a 2PT and a 3PT. Because two products are identical and a uniform per-unit price is applied to every unit from each manufacturer, the retailer will buy *all* of its demand from supplier i *only*, with surplus as $\max\{v(w_A), v(w_B)\}$ under LP, or as $\max\{v(w_A) - T, v(w_B)\}$ under a 2PT. It is this very all-or-nothing feature of the retailer's decision that drives the follower to always undercut the leader's offer and capture the whole market, as long as the per-unit price is above cost. Consequently, when two products are perfect substitutes, no matter whether it is LP or a 2PT adopted by the leader, the equilibrium of this Stackelberg game is that both manufacturers earn zero profit from marginal cost pricing.

Proposition 1 (LP and 2PT Equilibrium for the Case of Homogeneous Products) *When two products are perfect substitutes, the LP and 2PT equilibrium outcomes are the same as the simultaneous Bertrand outcome—–both manufacturers set prices at marginal cost and earn zero profits.*[12]

As explained above, this Bertrand paradox is a direct result from the "applying-to-all-units" feature of a single per-unit price in LP or a 2PT. Compared with either LP or a 2PT, the distinct element of a 3PT is its quantity threshold Q_o. The introduction of such a quantity threshold makes setting two different prices for different quantity ranges possible, as the marginal price is zero for quantity within the threshold and positive for that exceeding it. As shown in the next proposition, with a 3PT, both manufacturers set *above-cost* prices and earn *strictly positive* profits. This is in stark contrast with LP or 2PT equilibrium, and the Bertrand paradox is resolved.

Proposition 2 (3PT Equilibrium for the Case of Homogeneous Products) *When two products are perfect substitutes, under Assumptions 1~4, 3PT equilibrium* $(T_o^*, Q_o^*, w_A^*; w_B^*)$ *exists and is uniquely characterized by*

$$T_o^* = \widehat{w}_B^* Q_o^* \tag{1}$$

$$w_B^* \leq w_A^* \tag{2}$$

$$Q_o^* = \pi^{m\prime}(w_B^*), \tag{3}$$

where

$$(w_B^*, \widehat{w}_B^*) = \arg\max_{\substack{(x,y)\\ y<x}} \left\{ \begin{array}{c} (y-c)\pi^{m\prime}(x) \\ s.t. \pi^m(y) = \pi^m(x) - (x-c)\pi^{m\prime}(x) \end{array} \right\}. \tag{4}$$

In this equilibrium, both manufacturers earn strictly positive profits in equilibrium.

[12] Here homogeneity drives $T = 0$ for the 2PT. In other words, the equilibrium 2PT degenerates to LP when two products are perfect substitutes.

Recall that in either LP or 2PT model, the uniform per-unit price applied to every unit from each manufacturer leads to an all-or-nothing purchase decision from the retailer, which provokes the undercutting from the follower. Because two products are identical here, the only way for the leader to earn positive profit is to avoid the undercutting from the follower, which is impossible under LP or a 2PT. However, with the quantity threshold Q_o and per-unit price for incremental demand w_A, the leader now can credibly commit itself to the level of supply at Q_o. In equilibrium, this is realized by setting its per-unit price for incremental demand w_A^* higher than manufacturer B's per-unit price w_B^*, as indicated by inequality (2) above. Facing such a 3PT, the follower has two options——undercutting and seizing the whole market with a lower price, or accommodating and serving the residual demand at a higher price. Because the leader will have no sale if the follower undercuts its offer, a leader will choose Q_o and T_o appropriately to induce an accommodation from a rational follower. This is represented by the constraint in the program (4) in Proposition 2, where the left hand side is the maximal monopoly profit from undercutting and the right hand side is the maximal profit from supplying residual demand.[13]

Formally, this is a sequential-move game with complete information, we can solve the game by backward induction. It also turns out that the determination of the leader's optimal 3PT can eventually be reduced to a contract design problem. In particular, by carefully choosing the quantity threshold along with the per-unit price for overage, the leading firm can credibly commit to the market coverage at which it wants to serve and leave sufficient residual demand and profit to the follower, which prevents it from undercutting and triggering a price war.

Here we only sketch the proof, leaving the complete analysis to Appendix.1.

We first examine the monopoly retailer's problem in the last stage of the game. With two offers on the table, the retailer has two options——(AA): accepting A's 3PT, or (NA): rejecting A's 3PT. Let $p(q) \equiv R'(q)$ denote the marginal price implied by $R(q)$. We next write the highest price manufacturer B can charge for a positive sale when the retailer accepts manufacturer A's 3PT as $w_A^e \equiv \min\{w_A, p(Q_o)\}$. The residual demand for manufacturer B's products after buying Q_o units from A is then written as $q^r(w; Q_o) \equiv \arg\max_{y \geq 0}[R(Q_o+y) - w \cdot y] = 1\{w < w_A^e\} \cdot [q^m(w) - Q_o]$. In (AA), the manufacturer B must set $w_B < w_A^e$ in order to have a positive sale. The retailer's optimal purchase decision can be summarized in Figure 3.

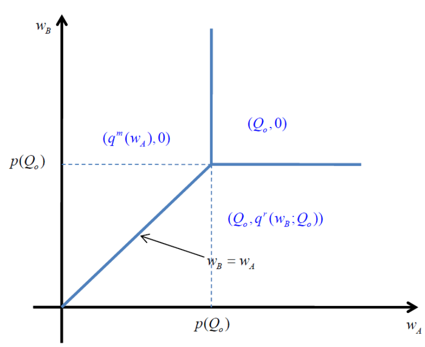


Figure 3: The Retailer's Optimal Purchase Decision in (AA)

After exploiting the properties of the retailer's profit curves in (AA) and (NA), we can prove that, in equilibrium, there must exist a unique cutoff $\widehat{w}_B = \frac{T_o}{Q_o}$ such that the retailer will reject A's

[13] Since the leader can always leave the follower an arbitrarily greater profit to induce a favorable response from the follower via a lower Q_o, we can, without loss of generality, infer that the follower will accommodate when the constraint in the program (4) above is binding. The same logic can be applied to the constraint in the program (10) in Proposition 5.

offer and buy exclusively from B if $w_B \leq \widehat{w}_B$, whereas accepting A's 3PT if $w_B > \widehat{w}_B$. Note that the cutoff $\widehat{w}_B = \frac{T_o}{Q_o}$ can be viewed as the average price for the first Q_o units from A.

This basically tells us that manufacturer B can choose to be either a monopoly supplier and earn $\pi^m(w_B)$ by setting its w_B below the cutoff $\widehat{w}_B$, or a residual demand supplier and earn residual profit $\pi^r(w_B; Q_o) \equiv (w_B - c)q^r(w_B; Q_o) = 1\{\widehat{w}_B < w_B < w_A^e\} \cdot [\pi^m(w_B) - (w_B - c) \cdot Q_o]$ by setting its w_B above the cutoff $\widehat{w}_B$ but below w_A^e. From the properties of $\pi^m(w_B)$ and $\pi^r(w_B; Q_o)$, we know that there is a discontinuous drop at $\widehat{w}_B$ in manufacturer B's profit curve, which is shown as the red curve in Figure 4.

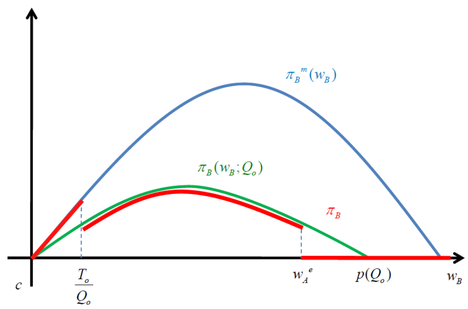


Figure 4: Manufacturer B's Profit Curve in Equilibrium

From its profit curve, we can easily see the trade-off manufacturer B faces: undercutting A's average price $\widehat{w}_B = \frac{T_o}{Q_o}$ for Q_o units can allow B to capture the whole market and make itself a monopoly supplier, but at a lower price; accommodating allows B to charge a higher price $\widehat{w}_B < w_B < w_A^e$, but at the cost of leaving Q_o units to manufacturer A. Let w_B^* be manufacturer B's optimal price in equilibrium.

We now turn to manufacturer A's choice of a 3PT. Manufacturer A's profit is

$$\pi_A = 1\{\widehat{w}_B < w_B^* < w_A^e\} \cdot (\widehat{w}_B - c)Q_o + 1\{w_A^e \leq w_B^*\} \cdot [(\widehat{w}_B - w_A^e)Q_o + (w_A^e - c)q^m(w_A^e)].$$

We know that manufacturer B would never choose $w_A^e \leq w_B$, because it would earn zero in that case. Thus, for possible positive profit, manufacturer A must ensure $\widehat{w}_B < w_B^* < w_A^e$. And this is equivalent to $\max_{x \leq \widehat{w}_B} \pi^m(x) \leq \pi^r(w_B^*; Q_o)$, which says being a residual demand supplier is at least as profitable as being a undercutting monopoly. So manufacturer A's problem can be written as

$$\max_{T_o, Q_o, w_A} (\widehat{w}_B - c)Q_o$$

$$s.t. \max_{x \leq \widehat{w}_B} \pi^m(x) \leq \pi^r(w_B^*; Q_o) \tag{5}$$

$$w_B^* = \arg\max_{\widehat{w}_B < x < w_A^e} \pi^r(x; Q_o) \tag{6}$$

$$\widehat{w}_B = \frac{T_o}{Q_o}. \tag{7}$$

Note that the whole game now is reduced to a contract design problem from manufacturer A's point of view. Constraint (5) is equivalent to an incentive-compatibility constraint in the standard contract design problem. Constraint (6) is the definition of w_B^*, and constraint (7) is the characterization of the cutoff point condition from retailer's optimal choice. At the optimum, the incentive-compatibility constraint holds with equality. By substituting the constraint into manufacturer A's profit function, it is equivalent for manufacturer A to set $(w_B^*, \widehat{w}_B^*)$, and the problem is characterized by the program (4) in Proposition 2.

By restricting its own supply to Q_o, it makes accommodation more profitable for the follower than undercutting for the follower. First, a small average price $\widehat{w}_B = \frac{T_o}{Q_o}$ for Q_o units from A makes the cost of undercutting large. Second, a small Q_o limits the loss of demand from accommodation. These two forces resolve the Bertrand paradox and soften competition.

■ **Linear Demand Example** For general linear demand $q(w) = \alpha - w$ generated from $R(q) = q(\alpha - \frac{1}{2}q)$ with cost c, denote the term standing for efficiency level as $\lambda \equiv \alpha - c$, the equilibrium is $T_o^* = [\frac{\lambda}{2} \cdot (1 - \frac{\sqrt{2}}{2}) + c] \cdot \lambda \cdot (1 - \frac{\sqrt{2}}{2}), Q_o^* = \lambda \cdot (1 - \frac{\sqrt{2}}{2}), w_A^* \geq w_B^* = \frac{\lambda}{2} \cdot \frac{\sqrt{2}}{2} + c, \widehat{w}_B = \frac{\lambda}{2} \cdot (1 - \frac{\sqrt{2}}{2}) + c$, and manufacturers' profits are $\pi_A = \frac{\lambda^2}{2} \cdot (\frac{3}{2} - \sqrt{2}), \pi_B = \frac{\lambda^2}{2} \cdot \frac{1}{4}$, and retailer's profit is $\pi_R = \frac{\lambda^2}{2} \cdot (\sqrt{2} - \frac{7}{8})$.[14]

Notice that the average price for Q_o units from manufacturer A $\widehat{w}_B^*$ is strictly lower than w_B^*—–the lower bound of its per-unit price for incremental demand. This reflects that the equilibrium 3PT involves a quantity premium, such that it would cost the retailer more per unit for purchases exceeding Q_o units than those within Q_o units from manufacturer A. As will be shown later, the equilibrium 3PT does not have to involve a quantity premium, especially when two products are more differentiated.

3.2 The Case of Imperfect Substitutes

We now extend the model to the case in which two competing products from A and B are imperfect substitutes. This is an important case to consider because product differentiation is a feature of real life markets. In addition, the homogeneity of the manufacturers' products is crucial for the Bertrand paradox in the last subsection. One may wonder how general our results on a 3PT can be when products are differentiated. As we shall see, with any degree of product substitutability, a 3PT always helps the leader in terms of quantity sale and profit as in the perfect substitutes case, but it may *not* soften competition and benefit the rival firm as in the case of homogeneous products. Rather, a 3PT could intensify competition and have some exclusionary effects on the rival as two manufacturers' products become less substitutable.[15] We will first characterize the 3PT equilibrium in this subsection, and then we will explore its welfare effects in the next section.

With product differentiation, now the retailing revenue function is a general function $R(q_A, q_B)$. Write the retailer's profit when facing per-unit prices (w_A, w_B) as $v(w_A, w_B) \equiv \max_{x\geq 0, y\geq 0}[R(x,y) - w_A \cdot x - w_B \cdot y]$, and its optimal quantities as $(q_A(w_A, w_B), q_B(w_A, w_B)) \equiv \arg\max_{x\geq 0, y\geq 0}[R(x,y) - w_A \cdot x - w_B \cdot y]$. We use $\pi_i(w_A, w_B) \equiv (w_i - c)q_i(w_A, w_B), i = A, B$ to denote the manufacturer i's profit from a per-unit price charge under (w_A, w_B). We define the monopoly quantity faced by manufacturer B as $q_B^m(w) \equiv \arg\max_{y\geq 0}[R(0,y) - w \cdot y]$, and similarly for $q_A^m(w)$. If manufacturer B is the sole supplier for the retailer, it will earn monopoly profit $\pi_B^m(w) \equiv (w - c)q_B^m(w)$. Denote the residual demand for manufacturer B's products after buying Q_o units from A as $q_B^r(w; Q_o) \equiv$

[14]Since in this example $\pi_A < \pi_B$, one may wonder why firm B earns a higher profit while firm A is the dominant firm in our model. We can consider the competition modelled in the article is only on the "contestable" part of the market. Thus, π_A here is only part of firm A's profit. Other than π_A, firm A has another profit channel from its "captive" market which is not exposed to competition and we abstract away from. Furthermore, even in such "contestable" part of the market only, we will have $\pi_A > \pi_B$ as two products become more differentiated. This is shown in an earlier version of this article (see Chao, 2010).

[15]Here we measure the level of competition in terms of efficiency. In particular, the closer the total surplus toward the efficient level, the more intensified the competition is.

$\arg\max_{y\geq 0}[R(Q_o, y) - w \cdot y]$, and write its profit from being a residual demand supplier as $\pi^r_B(w; Q_o) \equiv (w-c)q^r_B(w; Q_o)$. Let $p_A(q_A, q_B) \equiv R_A(q_A, q_B)$ be the marginal price for product A implied by $R(q_A, q_B)$. Similarly we can define $p_B(q_A, q_B)$.

Corresponding to the assumptions in the case of homogeneous products, we make the following parallel assumptions here.

Assumption 5 (Monotonicity and Diagonal Dominance) *$R(q_A, q_B)$ is $\mathbb{C}^2$ on $\mathbb{R}^2_+$. $\forall q_j \geq 0$, as long as $\widehat{q}_i(q_j) > 0$, we have $R_i > 0, \forall q_i \in [0, \widehat{q}_i(q_j))$; $R_{ii} < R_{ij} < 0, \forall (q_A, q_B) \in \mathbb{R}^2_+, i, j = A, B, i \neq j$. Here $\widehat{q}_i(q_j)$ is the finite satiation quantity on i when q_j is bought. That is, $\widehat{q}_A(q_B) \equiv \arg\max_{q_A\geq 0} R(q_A, q_B), \forall q_B \geq 0$; $\widehat{q}_B(q_A) \equiv \arg\max_{q_B\geq 0} R(q_A, q_B), \forall q_A \geq 0$.*

Assumption 5 says the retailing revenue function is increasing in each argument, and the own effect on marginal revenue is larger than the cross effect on it. This guarantees the strict concavity of $R(q_A, q_B)$. Denote the regions $D_1 \equiv \{(w_A, w_B)\, | q_A(w_A, w_B) > 0, q_B(w_A, w_B) > 0\}$ and $D_2 \equiv \{(w, Q_o)\, | q^r_B(w; Q_o) > 0|\}$. Assumption 5 implies that both D_1 and D_2 are non-empty. In addition, $q_i(w_A, w_B)(i = A, B)$ is a well-defined continuously differentiable function with $0 < \frac{\partial q_i(w_A,w_B)}{\partial w_j} < \left|\frac{\partial q_i(w_A,w_B)}{\partial w_i}\right| = -\frac{\partial q_i(w_A,w_B)}{\partial w_i}, \forall (w_A, w_B) \in D_1$; and $q^r_B(w; Q_o)$ is a well-defined continuously differentiable function with $\frac{\partial q^r_B(w;Q_o)}{\partial w} < 0, \frac{\partial q^r_B(w;Q_o)}{\partial Q_o} < 0, \forall (w, Q_o) \in D_2$. Moreover, Assumption 5 ensures that $\pi_i(w_A, w_B)$ and $\pi^r_B(w; Q_o)$ are $\mathbb{C}^1$, $\forall (w_A, w_B) \in D_1$ and $\forall (w, Q_o) \in D_2$, respectively.

Assumption 6 (Efficiency Requires Carrying Both Products) $(c, c) \in D_1$.

Assumptions 5 and 6 ensure $q_i(c, c) \in (0, \infty), i = A, B$. That is, there is no exclusion in efficiency.

Assumption 7 (Concavity and Single-Crossing of Profit Functions) *(i)* $-\frac{\partial^2 \pi_B(w_A,w_B)}{\partial w_B^2} \geq \frac{\partial^2 \pi_B(w_A,w_B)}{\partial w_B \partial w_A} > 0, \forall (w_A, w_B) \in D_1$;

(ii) $\frac{\partial^2 \pi_B(w_A,w_B)}{\partial w_A^2} \geq \frac{\partial^2 \pi_B(w_A,w_B)}{\partial w_B \partial w_A} \cdot \frac{\frac{\partial^2 \pi_B(w_A,w_B)}{\partial w_B \partial w_A}}{\frac{\partial^2 \pi_B(w_A,w_B)}{\partial w_B^2}}, \forall (w_A, w_B) \in D_1$;

(iii) $\frac{\partial^2 \pi^r_B(w_B;Q_o)}{\partial w_B^2} < 0, \frac{\partial^2 \pi^r_B(w_B;Q_o)}{\partial w_B \partial Q_o} < 0, \forall (w_B, Q_o) \in D_2$.

This assumption essentially represents the substitutability between A and B. (i) says B's profit function when A sets w_A is concave in w_B, and B's marginal profit increases with A's price. Moreover, the own-price effect is larger than the cross-price effect on its marginal profit. (ii) holds automatically when $\frac{\partial^2 \pi_B(w_A,w_B)}{\partial w_A^2} \geq 0$ since RHS is negative. Combining with (i), (ii) indicates that whenever $\frac{\partial^2 \pi_B(w_A,w_B)}{\partial w_A^2} < 0$, we have the own-price effect is larger than the cross-price effect on B's cross marginal profit, that is, $\left|\frac{\partial^2 \pi_B(w_A,w_B)}{\partial w_A^2}\right| \leq \frac{\partial^2 \pi_B(w_A,w_B)}{\partial w_B \partial w_A} \cdot \left|\frac{\frac{\partial^2 \pi_B(w_A,w_B)}{\partial w_B \partial w_A}}{\frac{\partial^2 \pi_B(w_A,w_B)}{\partial w_B^2}}\right| \leq \frac{\partial^2 \pi_B(w_A,w_B)}{\partial w_B \partial w_A}$. (iii) states that B's profit function when A sets Q_o is concave in w_B, and B's marginal profit decreases with A's quantity. These assumptions are mainly for our comparative statics results. It is easy to verify that general differentiated linear demands satisfy this assumption.

As before, we begin with the benchmark case where the leader can only offer a linear price. Next, we look at the 2PT equilibrium as an improvement over LP, and after that, we characterize the 3PT equilibrium as a further improvement over a 2PT.

When two products are differentiated, the retailer does not simply buy the product from the cheaper source as in the case of homogeneous products. Under LP, the retailer will decide whether to carry both products and earn $v(w_A, w_B)$ or buy exclusively from manufacturer B and earn $v(\infty, w_B)$. It is easy to see that the former weakly dominates the latter. And the equilibrium is as follows.

Proposition 3 (LP Equilibrium for the Case of Differentiated Products) *When two products are imperfect substitutes, the LP equilibrium is* (w_A^{LP}, w_B^{LP})*, where*
(i) $w_B^{LP} = B(w_A^{LP}) = \arg\max_w \pi_B(w_A^{LP}, w)$;
(ii) $w_A^{LP} = \arg\max_w \pi_A(w, B(w))$.

This is the classical Stackelberg price-setting equilibrium, with $B(w_A)$ as the follower's optimal response function for any given w_A. It can be found in Gal-Or (1985).

With sequential move, the follower's response function remains the same as that with simultaneous move, while the leader now can take into account the follower's response when making its own decision. Furthermore, this sequential moving will facilitate the leader to set the tone for competition. In such a price-setting game, the follower enjoys a second-mover advantage, and both manufacturers are still better off than that in the simultaneous-move case.[16] Consequently, if we allow manufacturers choose to move simultaneously or sequentially, then they both will agree with the latter one.[17]

Corollary 1 (Sequential-move Helps Both Manufacturers Over Simultaneous-move) *(1)* $\pi_i^{LP} > \pi_i^{\textit{Simultaneous\_LP}}$; *(2)* $\pi_i^{2PT} > \pi_i^{\textit{Simultaneous\_2PT}}$; *(3)* $\pi_i^{3PT} > \pi_i^{\textit{Simultaneous\_3PT}}$; $i = A, B$.

When 2PT is available for the leading firm, the fixed fee T can help the leading firm to extract more surplus. Now, the retailer will earn $v(w_A, w_B) - T$ if buying both, but $v(\infty, w_B)$ if buying from B only. Then, parallel to the reasoning process in the case of homogeneous products, we can show that, in equilibrium, they must cross each other only once at $\widehat{w}_B^{2PT}$ and $T = v(w_A, \widehat{w}_B^{2PT}) - v(\infty, \widehat{w}_B^{2PT})$. In that case, the equilibrium is as follows.

Proposition 4 (2PT Equilibrium for the Case of Differentiated Products) *When two products are imperfect substitutes, the 2PT equilibrium is* $((T^{2PT}, w_A^{2PT}), w_B^{2PT})$*, where*
(i) $w_B^{2PT} = B(w_A^{2PT}) = \arg\max_w \pi_B(w_A^{2PT}, w)$;
(ii) $(w_A^{2PT}, \widehat{w}_B^{2PT}) = \arg \max\limits_{\substack{(w,\widehat{w}) \\ \widehat{w}<B(w)}} \left\{ \begin{array}{c} \pi_A(w, B(w)) + v(w, \widehat{w}) - v(\infty, \widehat{w}) \\ s.t. \pi_B(w, B(w)) = \pi_B^m(\widehat{w}) \end{array} \right\}$;

[16] For discussion of the first-mover and the second-mover advantage in Stackelberg model, see Gal-Or (1985), Amir and Stepanova (2006).

[17] One may wonder why the dominant firm wants to move first in the presence of the second-mover advantage. First, this leadership can arise simply because of its dominance, as noted by Markham (1951, pp. 895)—–"*Price 'leadership' in a dominant firm market is not simply a modus operandi designed to circumvent price competition but is instead an inevitable consequence of the industry's structure*". Second, the endogenous price leadership literature (Deneckere and Kovenock, 1992, van Damme and Hurkens, 2004) provides several game-theoretical models justifying the dominant firm's leadership when the follower enjoys the second-mover advantage. I thank an anonymous referee for pointing this out.

(iii) $T^{2PT} = v(w_A^{2PT}, \widehat{w}_B^{2PT}) - v(\infty, \widehat{w}_B^{2PT})$.
In this equilibrium, the leader earns higher profit than that under LP.

Parallel to the case of homogeneous products, we will see how a 3PT can further improve the leader's profit over LP or a 2PT. Similar to Proposition 2, we have the following proposition. The complete analysis is available in Appendix.2.

Proposition 5 (3PT Equilibrium for the Case of Differentiated Products) *When two products are imperfect substitutes, under Assumptions 5~7, 3PT equilibrium* $(T_o^*, Q_o^*, w_A^*; w_B^*)$ *exists and is characterized by*[18]

$$T_o^* = \max_y [R(Q_o^*, y) - \widehat{w}_B^* y] - v(\infty, \widehat{w}_B^*) \quad (8)$$

$$\underline{w}_A \equiv p_A(Q_o^*, q_B^r(w_B^*; Q_o^*)) \leq w_A^*, \quad (9)$$

where

$$(Q_o^*, w_B^*, \widehat{w}_B^*) = \arg \max_{\substack{(q,x,y) \\ y<x}} \left\{ \begin{array}{c} \max_z [R(q,z) - y \cdot z] - v(\infty, y) - c \cdot q \\ s.t. \pi_B^m(y) = \pi_B^r(x; q) \\ \frac{\partial \pi_B^r(x;q)}{\partial w_B} = 0 \end{array} \right\}. \quad (10)$$

In this equilibrium, the leader earns higher profit than that under LP or a 2PT.

More interestingly, the basic idea of utilizing a 3PT as a credible commitment to induce the competition toward the leader's interests generalizes to any degrees of substitutability, but with a new twist on competition—–now the degree of product differentiation interacting with the second-mover advantage plays a key role in the switch of a 3PT's competitive effects.

Starting from LP equilibrium, due to the presence of a fixed fee, a 2PT tends to reduce the per-unit price in order to extract surplus more efficiently. This downward pressure on the per-unit price will make the competition more severe as the rival manufacturer will respond in price cut. Nonetheless, this fiercer competition may not be in the best interests of the leader, especially when the second-mover advantage is very strong, as explained in the case of homogeneous products above. For this reason, the new element from a 3PT—–a quantity threshold—–can mitigate the second-mover advantage and thus improve over a 2PT under competition. As the intuition from the case of perfect substitutes suggests, when two products are more substitutable, the second-mover advantage is so strong that the 3PT would be used to restrict competition. Instead, as two products become more and more differentiated, the second-mover advantage is diluted, and the 3PT can function more as a surplus extraction tool toward a 2PT. This can be seen from the other extreme case—–when two products are independent, the 3PT equilibrium outcome will converge to the 2PT's efficient surplus extraction equilibrium outcome. From the continuity, we know that a 3PT must switch from a competition-softening tool to a competition-intensifying device as the degree of product differentiation increases. We will illustrate this twist in detail in the next section.

Before our comparative statics analysis, we have proved that the competition is most intensified under a 2PT than under either LP or a 3PT.

[18] We do not establish uniqueness of the 3PT equilibrium for any general revenue function. Such a result would require assumptions that are more restrictive than Assumptions 5~7. However, we have shown uniqueness of the 3PT equilibrium for differentiated linear demand system in Section 4, before we perform comparative statics analysis.

Proposition 6 (2PT Intensifies Competition Most) *When two products are imperfect substitutes, under Assumptions 5~7,*

(i) $w_A^{2PT} < \min\{\underline{w}_A, w_A^{LP}\}, w_B^{2PT} < \min\{w_B^*, w_B^{LP}\}$.
(ii) $\pi_A^{LP} < \pi_A^{2PT} < \pi_A^{3PT}, \pi_B^{2PT} < \min\{\pi_B^{3PT}, \pi_B^{LP}\}$.
(iii) $\pi_R^{3PT} < \pi_R^{2PT}$.
(iv) $\max\{TS^{3PT}, TS^{LP}\} < TS^{2PT}$.

Part (i) says the per-unit prices for both A and B under a 2PT are the lowest compared with those under either LP or a 3PT. An immediate implication of this is part (iv), as what matters for the efficiency here are the per-unit prices. The first part of (ii) is not surprising as more complicated pricing scheme helps the leading firm, and the second part indicates that a 2PT hurts manufacturer B most. This is because the fixed fee T in a 2PT provides an extra and more efficient channel to extract surplus, there is a downward pressure on the per-unit price. It turns out that such a downward pressure is so large that competition is most intensive under a 2PT, from which manufacturer B gets hurt most. Part (iii) shows that, compared with a 3PT, the more intensive competition under a 2PT benefits the retailer.

4 Comparative Statics

In the above section, we have characterized the 3PT equilibrium for general retailing revenue function $R(q_A, q_B)$, and we have shown that it improves the leading firm's profit over either LP or 2PT equilibrium. To illustrate our analysis above and gain more insights on how the product differentiation affects the equilibrium, in this section, we perform the comparative statics to investigate the effects of product differentiation on equilibrium.

We consider a general differentiated linear demand system, which is generated by the retailing revenue function

$$R(q_A, q_B) = \alpha(q_A + q_B) - \frac{1}{2}(q_A^2 + q_B^2 + 2\beta q_A q_B),$$

where $\alpha > 0$, $1 \geq \beta \geq 0$. The parameter β measures the degree of substitution between products A and B. The larger β is, the more substitutable two products are. When $\beta = 1$, two products are homogeneous. When $\beta = 0$, two products are independent. When $1 > \beta > 0$, two products are imperfect substitutes. The following proposition shows the existence and uniqueness of the 3PT equilibrium for this linear demand system.

Proposition 7 (Uniqueness of the 3PT Equilibrium for the Differentiated Linear Demands) *In the case of differentiated linear demands, that is, when* $R(q_A, q_B) = \alpha(q_A + q_B) - \frac{1}{2}(q_A^2 + q_B^2 + 2\beta q_A q_B)$*, the 3PT equilibrium exists and is uniquely characterized by Proposition 5.*

Now we can directly apply Propositions 3, 4 and 5 to compute the corresponding equilibrium. The results are listed in Tables 1 and 2 in Appendix.4.

A very nice property from those computed equilibria is that all the surplus functions (i.e. producer surpluses and total surpluses) can be written as products of a term standing for efficiency level, say $(\alpha - c)^2$, and a term only depending on the degree of substitution parameter β. Utilizing this property, we can compare all these surpluses without worrying about the efficiency level $(\alpha - c)^2$, which can be cancelled out when comparing. As will be shown below, all the cutoffs in this model can be uniquely identified based only on β.[19]

■ **Manufacturer B's Profit**

The equilibrium manufacturer B's profits under three regimes are depicted in Figure 5.

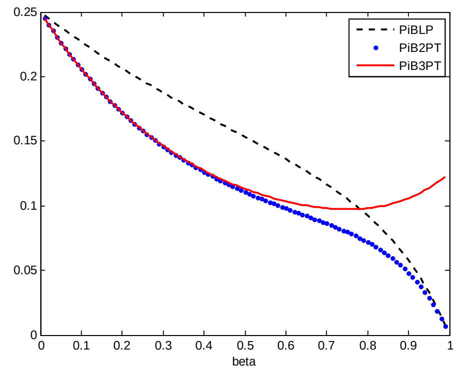


Figure 5: Manufacturer B's Profit under LP, 2PT, and 3PT Equilibrium

Proposition 8 *There exists a unique* $\beta_1 = 0.78$ *such that* $\pi_B^{3PT} \lesseqqgtr \pi_B^{LP}$ *when* $\beta \lesseqqgtr \beta_1$.

This is interesting because manufacturer B gets hurt when manufacturer A switches from LP to a nonlinear pricing——a 2PT, yet becomes better off when manufacturer A moves further to a more ornate nonlinear pricing——a 3PT.

The reason for this is that the introduction of a fixed fee from a 2PT gives manufacturer A a more efficient surplus extraction instrument. This more efficient surplus-extraction method puts a downward pressure on its per-unit price, which intensifies competition against manufacturer B, resulting in a lower profit for B than the one in LP equilibrium.

Under such competitive pressure, manufacturer B will price more aggressively as a follower. This is not in the best interests of the leader. Under a 3PT, the extra tool——the quantity target——actually allows manufacturer A to restrict its supply and mitigate the more aggressive response from the follower when its second-mover advantage is significant. This can be seen from the polar case, where two products are homogeneous. Thus, the quantity target in a 3PT can not only extract surplus in a similar vein as a 2PT, but can also be a commitment to mitigate the second-mover advantage if needed.

■ **Retailer R's Profit**

The equilibrium retailer R's profits under three regimes are shown in Figure 6.

[19] Consequently, we can drop the efficiency level term $(\alpha - c)^2$ when comparing these surpluses' relative magnitude. All the surplus functions shown in the figures below are only a function of β after cancelling out $(\alpha - c)^2$.

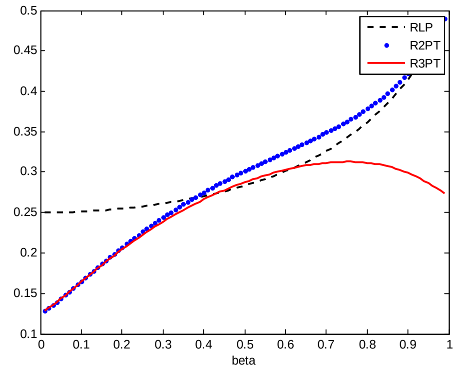


Figure 6: Retailer R's Profit under LP, 2PT, and 3PT Equilibrium

Proposition 9 ***(i)*** *There exists a unique* $\beta_2 = 0.37$ *such that* $\pi_R^{2PT} \lesseqqgtr \pi_R^{LP}$ *when* $\beta \lesseqqgtr \beta_2$.

(ii) *There exists a unique* $\beta_3 = 0.44$ *and* $\beta_4 = 0.62$ *such that* $\pi_R^{3PT} < \pi_R^{LP}$ *when* $\beta < \beta_3$; $\pi_R^{3PT} > \pi_R^{LP}$ *when* $\beta_3 < \beta < \beta_4$; *and* $\pi_R^{3PT} < \pi_R^{LP}$ *when* $\beta_4 < \beta$.

Compared with LP, although a 2PT will push manufacturer B to lower its per-unit price offer, the fixed fee can extract the surplus from the retailer more efficiently, and this may offset the competitive gain for the retailer from the lowered per-unit prices. Part (i) shows how the two forces balance. When two products are more homogeneous, the competition effect dominates the fixed fee effect. Nonetheless, when two products are more differentiated, the fixed fee extraction will offset the competitive gain from reduced prices.

The case for a 3PT is more complicated. First of all, it has the feature of a 2PT in extracting surplus more efficiently through the fixed fee, as well as the zero marginal price within the quantity threshold, which may provoke more aggressive response from the follower. Second, its commitment power of restricting its supply level by setting the quantity target low helps it to soften the rival's second-mover advantage. Therefore, when two products are quite differentiated, restricting supply becomes secondary because the second-mover advantage is diluted. A 3PT will then work more as a 2PT and extract surplus more efficiently from the retailer by the fixed fee. In this way, the retailer is worse off than in LP equilibrium. At the same time, when two products are very close substitutes, it will lessen competition by credibly committing to a limited supply. This limited supply induces the follower to accommodate rather than compete against the leader harshly. Hence, the retailer is worse off, too. Note that the retailer gets hurt in these two end cases, but for different reasons——in the former case, its surplus is extracted more by the fixed fee; in the latter case, a 3PT harms it by softening competition and preventing the follower from undercutting. Interestingly, in the middle range, when two products are neither too differentiated nor too homogeneous, the retailer can be better off than under LP equilibrium. This is the range in which the fixed fee has not been that efficient in surplus extraction because two products are not that differentiated, but the degree of substitution has not been large enough neither to justify a competition-softening strategy. In this case, competitive gains from the follower's more aggressive response dominate the fixed fee extraction effect and make the retailer better off.

■ **Total Surplus**

The equilibrium total surplus under three regimes are summarized in Figure 7.

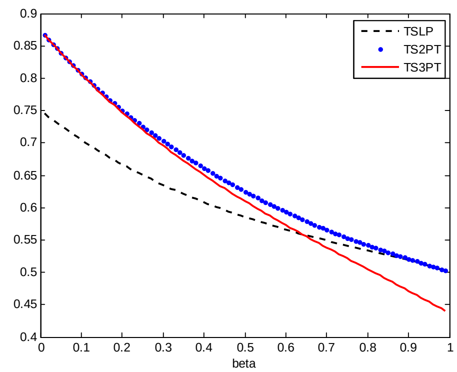


Figure 7: Total Surplus under LP, 2PT, and 3PT Equilibrium

Proposition 10 *There exists a unique* $\beta_5 = 0.64$ *such that* $TS^{3PT} \gtreqqless TS^{LP}$ *when* $\beta \lesseqqgtr \beta_5$.

Compared with LP, a 3PT will result in lower total surplus when two products are more substitutable, as it will soften competition due to the significant second-mover advantage then.

5 Extensions

5.1 When There is a Sunk Cost for Manufacturer B

From Proposition 6, we know that $\pi_B^{2PT} < \min\{\pi_B^{3PT}, \pi_B^{LP}\}$. Therefore, a 2PT hurts manufacturer B most. As illustrated in Proposition 8, the relationship between π_B^{3PT} and π_B^{LP} is more interesting——whether B earns more or less profit under a 3PT than that under LP depends on the degree of production differentiation between the two competing products. This striking difference between the competitive effects of a 2PT and those of a 3PT confirms that these two pricing schemes differ quite a lot, and it is worthwhile to further explore the 3PT's exclusionary effect, if any.

From the perspective of antitrust enforcement, the primary concern regarding a 3PT is its potentially exclusionary effect on rivals. Due to the lack of sunk cost or scale effects in production, the exclusion of B is assumed away in our previous analysis. In this subsection, we explore the potential risk of exclusion in more depth by introducing a sunk cost for the rival.[20] Sunk costs are present in many industries. For example, expenditures for R&D and marketing are usually considered as sunk investments, and most productions entail sunk costs in the form of capital equipment. In contrast to the incumbent who has already incurred these costs, the entrant will only produce if earnings exceed the sunk outlays. To reduce the risk associated with the sunk cost, the potential entrant has many options, including contracting with customers before making irreversible investments.

Accordingly, we modify the game a little bit by assuming that manufacturer B has to incur a sunk cost F in order to produce. But as before, B can still offer a contract to the retailer at date 2 before production. In particular, we assume that, from date 1 to date 3, all three parties will behave exactly the same as in our original model, ignoring the sunk cost and possibility of no entry from B.[21] The only change is a newly added stage——in date 4, taking the retailer's purchase decision as

[20]Due to this sunk cost, manufactuer B's average cost will be decreasing before reaching its minimum efficient scale. Thus, this sunk cost is one way to represent scale economies in production.

[21]In principle, the competing contracts as well as the retailer's purchase decision should depend on the range of sunk cost F, which will complicate the characterization of the 3PT equilibrium quite a lot. For instance, A may offer a limit-pricing 3PT. It is beyond the scope of this article to examine the implications of sunk cost on contracting in details.

given, B will produce if and only if it earns non-negative profit after incurring the sunk cost. Note that this setting is least favorable to exclusive equilibrium, since all three parties are assumed to behave as if it is impossible to exclude B in the first three stages. So, if we still can see an exclusion in this setting, then 3PT's exclusionary effects will only be strengthened when all parties' strategies can be contingent on sunk cost F and lead to possible exclusion. In other words, this simple setting can be regarded as a *lower bound* for possible exclusion.

In antitrust economics, promoting total surplus is perceived as the final goal of antitrust law.[22] In order to consider the welfare with the risk of potential exclusionary effects, we have to evaluate the total surplus. We use the same general differentiated linear demand system as that in Section 4, taking into account the potential exclusion of the rival. Note that we have $\pi_B^{3PT} < \pi_B^{LP}$ when $\beta < \beta_1 = 0.78$. To study the exclusionary effect of a 3PT, we focus on this range of β. To make the analysis interesting, we look at F in a range that manufacturer B will enter under LP equilibrium, but not under 3PT equilibrium. That is, we assume $F \in [\pi_B^{3PT}, \pi_B^{LP}]$, because otherwise either exclusion will never occur or always occur under both pricing schemes. Given $\pi_B^{3PT} < \pi_B^{LP}$, I compute both the lower bound TS_L^{LP} and upper bound TS_U^{LP} for total surplus under LP, corresponding to $F = \pi_B^{LP}$ and $F = \pi_B^{3PT}$, respectively.

Taking into account the sunk cost and its resulted exclusion under a 3PT, the total surplus from a 3PT with exclusion, TS_{ED}^{3PT}, changes due to A's 3PT and the loss of one product variety from B. In LP equilibrium, B will supply. The society enjoys one more product with a LP vs. LP competition, although it has to pay the sunk cost F. The equilibrium results are depicted below, in Figure 8.

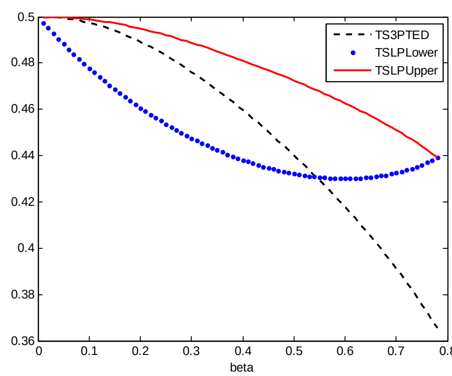


Figure 8: Total Surplus under 3PT Exclusion Equilibrium,
and Lower Bound and Upper Bound of Total Surplus under LP Equilibrium

Proposition 11 *When the entrant has to incur a sunk cost $F \in [\pi_B^{3PT}, \pi_B^{LP}]$, there exists a unique $\beta_6 = 0.54$, such that*

(i) *When $0 < \beta < \beta_6$, $TS_L^{LP} < TS_{ED}^{3PT} < TS_U^{LP}$. Exclusion occurs under a 3PT, and the efficiency under 3PT exclusion equilibrium may be higher or lower, depending on whether F is closer to the upper bound of π_B^{LP} or not.*

(ii) *When $\beta_6 < \beta < \beta_1$, $TS_{ED}^{3PT} < TS_L^{LP} < TS_U^{LP}$. Exclusion occurs under a 3PT, and the efficiency under 3PT exclusion equilibrium is lower than under LP equilibrium.*

(iii *When $\beta_1 < \beta$, $TS^{3PT} < TS^{LP}$. No exclusion, and a 3PT reduces efficiency over LP.*

[22] "*To an economist the thought of designing antitrust policy to maximize aggregate surplus comes naturally and, indeed, much of the economics literature implicitly has taken this to be the appropriate objective for antitrust policy*" (See Whinston, 2006, pp. 6-7).

Recall that in Proposition 10, where sunk cost is absent and exclusion is impossible, a 3PT is welfare reducing only if $\beta > \beta_5 = 0.64$. Nevertheless, with sunk cost and possible exclusion, as long as $\beta > \beta_6 = 0.54$, a 3PT must decrease welfare, and it is only *possible* for a 3PT to increase welfare when $\beta < \beta_6 = 0.54$. Clearly, in the presence of sunk cost and its resultant exclusion, the set of welfare-reducing outcomes from a 3PT is enlarged significantly. Hence, when sunk cost is significant, it is likely that a 3PT will be a barrier to entry and reduce welfare. Furthermore, with or without sunk cost and possible exclusion, a 3PT is more likely to reduce social welfare as two products become less differentiated. Thus, we should put a cautious eye on the 3PT, especially when there is a significant sunk cost or when two products are close substitutes.

5.2 When Manufacturer B Can Offer a 2PT or a 3PT

Thus far, we have confined our attention to the equilibrium when B can use LP only. This is consistent with our motivating example, *U.S. v. Microsoft Corp.*. The case is about Microsoft's business strategies for the sale of its MS-DOS operating system. The main competitor of MS-DOS then was Digital Research, Inc. (DRI)'s DR-DOS. DRI had never used a 3PT as Microsoft did; instead, it mainly adopted LP because OEMs felt MS-DOS is a must-carry product and they were reluctant to sign another 3PT with a small rival. Hence, DRI predominantly sold DR-DOS directly to the retail public and small- or medium-sized businesses.[23]

We now consider the extension to the same setting when B can offer a 2PT or a 3PT. This will help us understand the role of contract space restriction in our model.[24]

Proposition 12 (2PT vs. 2PT Equilibrium and 3PT vs. 3PT Equilibrium) *When both firms use a 2PT or a 3PT, the equilibrium outcomes are both efficient with surplus divisions as follows.*[25]

- ***2PT vs. 2PT:*** $\Pi_A^{2PT} = v(c,c) - v(\infty,c); \Pi_B^{2PT} = v(c,c) - v(c,\infty); \Pi_R^{2PT} = v(c,\infty) + v(\infty,c) - v(c,c)$;
- ***3PT vs. 3PT:*** $\Pi_A^{3PT} = v(c,c) - v(\infty,c); \Pi_B^{3PT} + \Pi_R^{3PT} = v(\infty,c)$, *where*

$$\begin{aligned} v(c,c) - v(c,\infty) &\leq \Pi_B^{3PT} \leq v(c,c) - [R(q_A(c,c),0) - c \cdot q_A(c,c)], \\ [R(q_A(c,c),0) - c \cdot q_A(c,c)] + v(\infty,c) - v(c,c) &\leq \Pi_R^{3PT} \leq v(c,\infty) + v(\infty,c) - v(c,c). \end{aligned}$$

When both firms can use a 2PT or a 3PT, the equilibrium outcome is always efficient, although the corresponding surplus divisions differ. Each manufacturer extracts its marginal contribution to

[23] See Baseman, Warren-Boulton and Woroch (1995), and "DRI Set to Ship DR-DOS 6.0 in Bid for Users of MS-DOS" on PC Week (Sep. 2nd, 1991).

[24] I am indebted to an anonymous referee for bringing this point to my attention.

[25] The surplus division between B and R varies with whether A offers a 3PT or not, and what kind of a 3PT is offered by A, although A's equilibrium profit will not change. In particular, if A chooses a 3PT with $Q_A = q_A(c,c), T_A = v(c,c) - v(\infty,c), w_A = \infty$, then B will get its upper bound profit as $v(c,c) - [R(q_A(c,c),0) - c \cdot q_A(c,c)]$ and R will get its lower bound profit as $[R(q_A(c,c),0) - c \cdot q_A(c,c)] + v(\infty,c) - v(c,c)$. By contrast, if A chooses a 3PT with $Q_A = 0, T_A = v(c,c) - v(\infty,c), w_A = c$, then B will get its lower bound profit as $v(c,c) - v(c,\infty)$ and R will get its upper bound profit as $v(c,\infty) + v(\infty,c) - v(c,c)$. Therefore, given a 2PT is available, A's 3PT plays a key role in the division of surpluses between B and R. If either B or R can form a coalition with A explicitly or implicitly, A's 3PT can easily get a Pareto improvement for the two parties in the coalition at the expense of the outsider. In short, a 3PT can do more than a 2PT can when B is allowed to offer a 2PT or a 3PT.

total surplus. This is consistent with common agency literature when two principals can *both* offer nonlinear contracts.[26] Nonetheless, it is in stark contrast with our previous equilibrium outcomes when B is restricted to LP——our 3PT equilibrium is never efficient, and the LP or 2PT equilibrium is efficient only when two products are homogeneous. Given that we do observe the 3PT in real antitrust cases while the traditional common agency literature does not provide a justification for it, this article identifies a scenario in which the 3PT has some bite.

The key difference between our model and those from common agency literature is the set of feasible contracts for B. In our model, B is restricted to offer LP only, while in the common agency literature two principals are usually assumed to use nonlinear contracts. Since LP cannot extract surplus efficiently as nonlinear contracts usually do, the restriction to LP creates contract externalities for the principals to reach the efficient outcome. Consequently, our model highlights that contract space restriction is very important to the strategic effects of a 3PT in competition.

6 Conclusion

3PTs have been commonly used for a long time, and nowadays, they are becoming even more popular in the information industry. In intermediate-goods markets, the use of a 3PT as a *vertical restraint* has become a hotly debated issue in the high-profile antitrust case *U.S. v. Microsoft Corp.*. The anticompetitive theory of a 3PT is based on the notion that the quantity threshold in the 3PT offered by a dominant firm, within which the marginal price is zero, will deprive the rival of the opportunity to reach minimum efficient scale. Nonetheless, the existing literature has thus far focused on interpreting the 3PT as a price discrimination tool.

In the absence of asymmetric information or downstream competition, we establish strategic roles of a 3PT under oligopoly and offer an equilibrium theory of a 3PT in a competitive context. We show that, compared with LP equilibrium and 2PT equilibrium, a 3PT always increases the leading firm's profit when competing against a rival with substitute products, in the absence of usual price discrimination or rent shifting motive. The distinct feature of a 3PT over a standard 2PT is its quantity threshold, which is the key provision being utilized to induce competition toward the leading firm's interest.

We establish further that product differentiation is a key determinant of the 3PT's function and welfare change. Under the general differentiated linear demand system, we show that the competitive effect of a 3PT in contrast to LP depends on the degree of substitutability between products——competition is *intensified* when two products are more differentiated, but *softened* when two products are more substitutable. This is in stark contrast with that of a 2PT, which always enhances competition and gives the highest total surplus of these three pricing schemes. Moreover, the rival firm always gets hurt in both profit and quantity sales when the dominant firm switches from LP to a 2PT, yet it may enjoy a higher profit when the dominant firm moves from a 2PT to the more ornate 3PT, although its quantity and market share are decreased even further.

In addition, we have shown that with sunk cost or scale economies, the 3PT can be a strategic barrier to entry and reduce social welfare. Thus, our results suggest that when sunk cost is significant or competing products are less differentiated, a 3PT is worrisome from an antitrust policy perspective.

The key assumption in our model is that manufacturer B is restricted to LP. When both manufacturers can use a 2PT or a 3PT, the equilibrium outcome is always efficient, and the 3PT is more

[26] See O'Brien and Shaffer (1997), Bernheim and Whinston (1998), and Marx and Shaffer (2004).

flexible than the 2PT in determining the division of surpluses between B and R. The more interesting case for the 3PT to have some bite is when B can only offer LP. We certainly recognize that any implications from our model may be limited due to the imposed asymmetries between two competing firms. Asymmetries between the dominant firm and its small competitors are not uncommon in many antitrust cases.[27] Those asymmetries can be incumbency, scale economies and restrictions on contracting. Our analysis provides a case showing how different contract spaces can result in rather different equilibrium outcomes in competition.

There are some other pricing schemes that could be considered as variations of a 3PT. One example is a two-block tariff (See Dolan, 1987). Another related pricing scheme is called all-units discount (See Kolay, Shaffer and Ordover, 2004, Chao and Tan, 2012). Given the common feature of a quantity target in all three of these pricing schemes, as well as the fact that, in practice, almost all the realized purchases under any of them are near or above the quantity threshold, both two-block tariff and all-units discounts can be converted to a 3PT, as indicated by the bold red dashed lines in Figure 9.

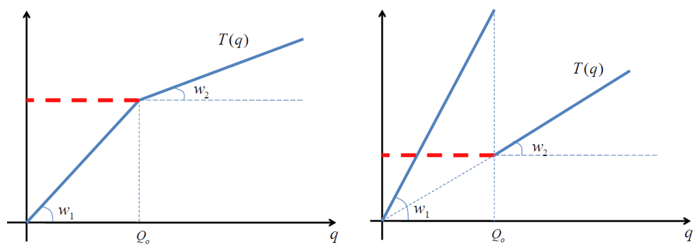


Figure 9: Two-Block Tariff and All-Units Discount

Therefore, our theory here can also be applied to these variations if their conversions to 3PTs approximate the original contracts well.

Appendix

Appendix.1 Equilibrium Analysis: The Case of Perfect Substitutes

The proof of Proposition 2 will follow from Lemmas 1 through 7.

■ Date 3: Retailer's Purchase Decision

Given (T_o, Q_o, w_A) from A and w_B from B, the retailer could either buy from A or not. Thus, we denote the retailer's two options as (AA): accepting A's 3PT and (NA): rejecting A's 3PT.

Lemma 1 (Retailer's Profit in (AA)) *In (AA), the retailer's profit function can be summarized as*[28]

$$r^{AA}(w_B; T_o, Q_o, w_A) = \begin{cases} v(w_B) + w_B Q_o - T_o & \text{if } w_B < w_A^e \\ v(w_A^e) + w_A^e Q_o - T_o & \text{if } w_A^e \leq w_B \end{cases} .$$

Proof of Lemma 1. In (AA), the retailer will in principle buy from both manufacturers, and its problem is as $r^{AA} = \max_{\eta \geq 0, q_B} [R(Q_o + \eta + q_B) - w_A \eta - w_B q_B] - T_o$. If $w_B < w_A^e$, then the retailer will stop buying from A after fulfilling Q_o requirement, and buy extra units from B. Then $r^{AA} = \max_{q_B} [R(Q_o + q_B) - w_B q_B] - T_o$. If $w_A^e \leq w_B$, then the retailer will buy exclusively from A. $r^{AA} = \max_{\eta \geq 0} [R(Q_o + \eta) - w_A \eta] - T_o$. ■

In (NA), the retailer will buy exclusively from manufacturer B. $r^{NA}(w_B) = v(w_B)$.

[27]When two firms are symmetric in every aspect, there won't be any antitrust concern, since each of them can simply match the other's strategies and neither would be disadvantaged.

[28]Graphically, the retailer's optimal purchase decision in (AA) is summarized in Figure 3 in the article.

Given such two options——(AA) or (NA), the retailer will choose the one giving him higher profit. The following lemma tells us the properties of the retailer's profit curves associated with these two options with respect to (w.r.t.) w_B.

Lemma 2 (Properties of r^{AA} and $\mathbf{r}^{NA}$) *(i) Slopes: (AA): $\frac{\partial r^{AA}}{\partial w_B} = 1\{w_B < w_A^e\} \cdot [Q_o - q^m(w_B)]$, (NA): $\frac{\partial r^{NA}}{\partial w_B} = -q^m(w_B)$; (ii) Cutoffs: $w_A^e \leq p(0)$; (iii) Relative steepness: $\frac{\partial r^{NA}}{\partial w_B} \leq \frac{\partial r^{AA}}{\partial w_B} \leq 0, \forall w_B$.*

Proof of Lemma 2. (i) follows directly from partial differentiation w.r.t. w_B. (ii) follows from the fact that $w_A^e \leq p(Q_o) \leq p(0)$ ($\because R''(q) < 0$). Because $-q^m(w_B) \leq Q_o - q^m(w_B), \forall w_B < w_A^e$ and $-q^m(w_B) \leq 0, \forall w_A^e \leq w_B$, (iii) follows. ■

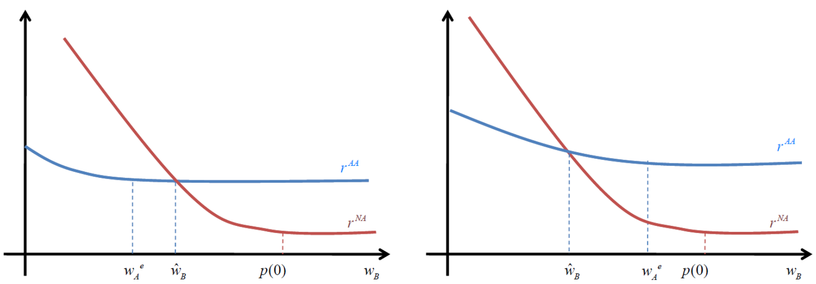
(i) tells us that r^{AA} and r^{NA} are both downward sloping before reaching a plateau w.r.t. w_B. From (ii), we know that the flat part of r^{AA} emerges before that of r^{NA}. (iii) states that r^{AA} is never steeper than r^{NA} for any given w_B.

From the properties of these profit curves, we know that there are only two possible cases: (r1) no-crossing: r^{NA} is always above $r^{AA}, \forall w_B$; (r2) single-crossing: r^{NA} crosses r^{AA} from above at $\widehat{w}_B$, where $r^{NA}(\widehat{w}_B) = r^{AA}(\widehat{w}_B; T_o, Q_o, w_A)$. (r1) is impossible in equilibrium because we are looking for some profit improvement from LP or 2PT equilibrium and in (r1) A would earn 0 profit for sure, which must be avoided by A if possible when designing the contract (T_o, Q_o, w_A) at date 1. Thus, the possible profit improvement can only occur in (r2).

Lemma 3 (Retailer's Choice) *In equilibrium, $\exists$ a unique $\widehat{w}_B(T_o, Q_o, w_A)$, where $r^{NA}(w_B) \gtreqless r^{AA}(w_B; T_o, Q_o, w_A)$ when $w_B \lesseqgtr \widehat{w}_B$.*

■ Date 2: Manufacturer B's Problem

From Lemma 3, we know that r^{NA} must cross r^{AA} uniquely from above at $\widehat{w}_B$. There are two possible cases of single-crossing: (B1) $w_A^e \leq \widehat{w}_B$; (B2) $\widehat{w}_B < w_A^e$, as shown in Figure A-1.



(B1) $w_A^e \leq \widehat{w}_B$ (B2) $\widehat{w}_B < w_A^e$

Figure A-1: Two Possible Cases of Crossings

B's profits and determinations of associated $\widehat{w}_B$ are summarized in Table A-1. In either (B1) or (B2), B will choose the optimal w_B^* to maximize π_B.

Table A-1: Manufacturers A and B's Profits under (B1) and (B2)

	A and B's Profits and Crossing Point $\widehat{w}_B$
(B1) $w_A^e \leq \widehat{w}_B$	$\pi_B = 1\{w_B \leq \widehat{w}_B\} \cdot \pi^m(w_B)$ (crossing point): $v(\widehat{w}_B) = v(w_A^e) + w_A^e Q_o - T_o$ $\pi_A = 1\{\widehat{w}_B < w_B^*\} \cdot [v(w_A^e) + (w_A^e - c)q^m(w_A^e) - v(\widehat{w}_B)]$
(B2) $\widehat{w}_B < w_A^e$	$\pi_B = 1\{w_B \leq \widehat{w}_B\} \cdot \pi^m(w_B) + 1\{\widehat{w}_B < w_B < w_A^e\} \cdot \pi^r(w_B; Q_o)$ (crossing point): $v(\widehat{w}_B) = v(\widehat{w}_B) + \widehat{w}_B Q_o - T_o$ $\pi_A = 1\{\widehat{w}_B < w_B^* < w_A^e\} \cdot (\widehat{w}_B - c)Q_o + 1\{w_A^e \leq w_B^*\} \cdot [(\widehat{w}_B - w_A^e)Q_o + (w_A^e - c)q^m(w_A^e)]$

■ Date 1: Manufacturer A's Problem

Accordingly, by substituting T_o using those in Table A-1, now A's profits are listed in Table A-1, too.

The following lemma shows that only (B2) is possible in equilibrium.

Lemma 4 *In equilibrium,* $c < \frac{T_o}{Q_o} = \widehat{w}_B < w_A^e$.

Proof of Lemma 4. $\frac{T_o}{Q_o} = \widehat{w}_B < w_A^e$: Suppose not. Then it would be (B1). In this case, the only possibility for A to earn some profit is that $\widehat{w}_B < w_B^*$, but B will get 0 then. As a result, for (B1) to be an equilibrium, A has to make $\widehat{w}_B < c$ so that $w_A^e \leq \widehat{w}_B < c = w_B^*$. Then $\pi_A = v(w_A^e) + (w_A^e - c)q^m(w_A^e) - v(\widehat{w}_B) < v(c) - v(\widehat{w}_B) < 0$, which contradicts with our objective of looking for positive profit.

$c < w_A^e$: Suppose not. Then $\widehat{w}_B < w_A^e \leq c = w_B^*$ and $\pi_A = (\widehat{w}_B - w_A^e)Q_o + (w_A^e - c)q^m(w_A^e) < 0$.

$c < \widehat{w}_B$: Suppose not. Then $\widehat{w}_B \leq c < w_B^* < w_A^e$ and $\pi_A = (\widehat{w}_B - c)Q_o \leq 0$. ■

■ Characterization of the Equilibrium

We begin with the properties of B's profit curves, which follows from direct computation.

Lemma 5 *(i)* $\pi^r(w; Q_o) \leq \pi^m(w), \forall w \geq c$ *with "=" at* $w = c$. *(ii)* $\frac{\partial \pi^r(w;Q_o)}{\partial w} \leq \pi^{m\prime}(w), \forall w \geq c, \forall Q_o > 0$. *(iii)* $\frac{\partial^2 \pi^r(w;Q_o)}{\partial w^2} \leq 0, \forall w \geq c$.

Consequently, π_B is given as the red line in Figure 4 in the article. From Table A-1, B would never choose $w_A^e \leq w_B$ because it would earn zero then. Table A-2 tells us that, for possible positive profit, A must ensure $\widehat{w}_B < w_B^* < w_A^e$. And this is equivalent to $0 < \max_{x \leq \widehat{w}_B} \pi^m(x) \leq \pi^r(w_B^*; Q_o)$, where the first inequality comes from $c < \widehat{w}_B$. The following lemma tells us the second inequality must be binding in equilibrium.

Lemma 6 *In equilibrium, (i)* $0 < \pi^{m\prime}(\widehat{w}_B)$ *and* $\max_{x \leq \widehat{w}_B} \pi^m(x) = \pi^m(\widehat{w}_B)$*; (ii)* $\widehat{w}_B < w_B^* < w_A^e$ *and* $\frac{\partial \pi^r(w_B^*;Q_o)}{\partial w_B} = 0$*; (iii)* $\pi^m(\widehat{w}_B) = \max_{x \leq \widehat{w}_B} \pi^m(x) = \pi^r(w_B^*; Q_o)$.

Proof of Lemma 6. (i): Suppose not. That is, $\pi^{m\prime}(\widehat{w}_B) \leq 0$. From Lemma 5, we know that $\frac{\partial \pi^r(\widehat{w}_B;Q_o)}{\partial w_B} \leq \pi^{m\prime}(\widehat{w}_B) \leq 0$, which implies $\max_{\widehat{w}_B < x < w_A^e} \pi^r(x; Q_o) = \pi^r(\widehat{w}_B; Q_o)$. However, $\pi^r(\widehat{w}_B; Q_o) < \pi^m(\widehat{w}_B), \forall \widehat{w}_B > c$ from Lemma 5, which contradicts with $\max_{x \leq \widehat{w}_B} \pi^m(x) \leq \pi^r(w_B^*; Q_o)$.

(ii): Because $\pi^r(\widehat{w}_B; Q_o) < \pi^m(\widehat{w}_B)$ from Lemma 5, in order for $\pi^r(w_B^*; Q_o) \geq \pi^m(\widehat{w}_B)$ to hold, we must have $\frac{\partial \pi^r(\widehat{w}_B;Q_o)}{\partial w_B} > 0$ and $\widehat{w}_B < w_B^*$. Moreover, because w_A^e does not enter the objective function, and we can increase it without any problem because $w_A^e \leq p(Q_o)$ holds by the definition of w_A^e. And $\pi^r(x; Q_o)\big|_{x=p(Q_o)} = 0$ and $\frac{\partial \pi^r(x;Q_o)}{\partial x}\big|_{x=p(Q_o)} < 0$ ensures that w_B^* is an interior solution for $\frac{\partial \pi^r(w_B^*;Q_o)}{\partial w_B} = 0$.

(iii): Suppose not. Notice that $\frac{\partial \pi_A}{\partial \widehat{w}_B} = Q_o \geq 0$, we can increase π_A by increasing $\widehat{w}_B$ until $\pi^r(w_B^*; Q_o) \geq \pi^m(\widehat{w}_B)$ becomes binding. ■

Consequently, A's problem can be rewritten as

$$\max_{T_o, Q_o, w_A} (\widehat{w}_B - c)Q_o$$
$$s.t.\pi^m(\widehat{w}_B) = \pi^m(w_B^*) - (w_B^* - c)Q_o$$
$$\pi^{m\prime}(w_B^*) = Q_o$$
$$\widehat{w}_B = \frac{T_o}{Q_o}$$

Besides, we need $w_B^* \leq w_A^e = \min\{w_A, p(Q_o)\}$. And $p(Q_o) > w_B^*$ automatically holds because $\pi^r(x; Q_o)\big|_{x=p(Q_o)} = 0$ and $\frac{\partial \pi^r(x;Q_o)}{\partial x}\big|_{x=p(Q_o)} < 0$. Thus, we only need $w_B^* \leq w_A$. Note that w_A does not enter A's objective function, and we always can guarantee $w_B^* \leq w_A$ holds by setting w_A sufficiently high. Meanwhile, (T_o, Q_o) can be uniquely determined by $\widehat{w}_B = \frac{T_o}{Q_o}$ and $\pi^{m\prime}(w_B^*) = Q_o$. So instead of choosing (T_o, Q_o), it is equivalent for A to set $(w_B^*, \widehat{w}_B^*) \in [c, w^m] \times [c, w^m]$ s.t.

[P] $$(w_B^*, \widehat{w}_B^*) = \underset{\substack{(x,y) \\ y<x}}{\operatorname{argmax}} \left\{ \begin{array}{c} (y-c)\pi^{m\prime}(x) \\ s.t.\pi^m(y) = \pi^m(x) - (x-c)\pi^{m\prime}(x) \end{array} \right\}.$$

Lemma 7 (Existence and Uniqueness) *Under Assumptions 1~4, there exists a unique interior solution* $(w^*, \widehat{w}^*) \in (c, w^m) \times (c, w^m)$ *to the program [P].*

Proof of Lemma 7. Let's first show that there exists a continuously differentiable function $y(x)$ satisfying the constraint

$$\pi^m(y(x)) = \pi^m(x) - (x-c)\pi^{m\prime}(x) = \pi^m(x) - h(x). \tag{A.1}$$

$\forall w \in (c, w^m), \pi^m(w) - h(w) < \pi^m(w)$. Besides, $\pi^{m\prime}(w) - h'(w) = -(w-c)\cdot\pi^{m\prime\prime}(w) > 0$, $\forall w \in (c, w^m)$ and $\pi^{m\prime}(w) > 0, \forall w \in (c, w^m)$. So both $\pi^m(w) - h(w)$ and $\pi^m(w)$ are strictly increasing in $w, \forall w \in (c, w^m)$. Combining these with the fact that at $w = c$ and $w = w^m$ we have $\pi^m(w) = \pi^m(w) - h(w)$, we can infer that, starting from c, $\pi^m(w) - h(w)$ must cross $\pi^m(w)$ from below at w^m as shown in Figure A-2 below.

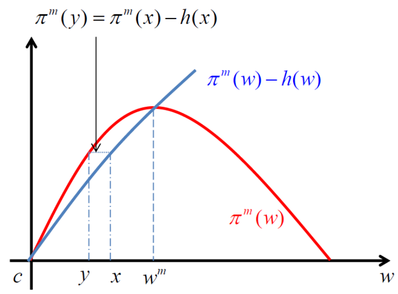


Figure A-2: $\pi^m(w) - h(w), \pi^m(w)$ and
the existence of the solution path for $y = y(x)$ s.t. $\pi^m(y(x)) = \pi^m(x) - h(x)$

We can easily see from Figure A-2 that $\forall x \in (c, w^m)$, $\exists$ a continuous differentiable function $y(x) \in (c, x)$ s.t. $\pi^m(y(x)) = \pi^m(x) - h(x)$. Further, $y'(x) = \frac{(x-c)[-\pi^{m\prime\prime}(x)]}{\pi^{m\prime}(y(x))} > 0$. Hence, (A.1) has a unique increasing solution path $y = y(x)$ from (c, c) to (w^m, w^m) as the red line in Figure A-4.

Now we can substitute $y = y(x)$ into A's objective function, so A's problem becomes

$$\max_x \,[y(x) - c]\cdot\pi^{m\prime}(x).$$

Denote A's objective function as $f(x) \equiv [y(x) - c]\cdot\pi^{m\prime}(x)$. It is easy to see $f(x)$ is continuous and differentiable, $\forall x \in (c, w^m)$. Then $f'(x) = \frac{(x-c)[-\pi^{m\prime\prime}(x)]}{\pi^{m\prime}(y(x))}\cdot\pi^{m\prime}(x) + [y(x) - c]\cdot\pi^{m\prime\prime}(x) = [\frac{-\pi^{m\prime\prime}(x)}{\pi^{m\prime}(y(x))}]\cdot[h(x) - h(y(x))]$. Note that $\frac{-\pi^{m\prime\prime}(x)}{\pi^{m\prime}(y(x))} > 0 (\because y(x) < x < w^m)$. Therefore, the sign of $f'(x) =$ the sign of $h(x) - h(y(x))$.

Now let's study the property of $h(w)$. Recall that Assumption 4 ensures that $h(w) = (w-c)\pi^{m\prime}(w)$ is single-peaked in $[c, w^m]$. Denote the value at which the single peak occurs as $w^{**} \equiv \arg\max_w h(w)$. Thus, we must have $h'(w) > 0, \forall c \leq w < w^{**}$, and $h'(w) < 0, \forall w^{**} < w < w^m$. Combining with the fact that $h(c) = h(w^m) = 0$, we know that $h(w)$ must be an inverted U-shaped curve as shown in Figure A-3 below.

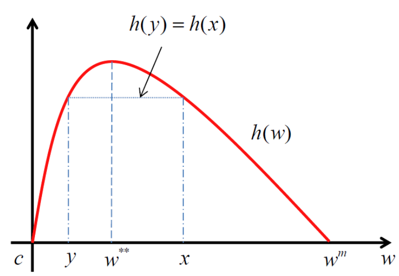


Figure A-3: $h(w)$ and
the existence of solution path $y = z(x)$ s.t. $h(z(x)) = h(x)$

It is easy to see that $\forall x \in (w^{**}, w^m)$, $\exists$ a continuously differentiable function $y = z(x) \in (c, w^{**})$ s.t. $h(z(x)) = h(x) > 0$. Further, $z'(x) = \frac{h'(x)}{h'(z(x))} < 0, \forall z(x) < w^{**} < x$. Therefore, $h(y) = h(x)$ must have a unique downward sloping solution path $y = z(x)$ starting from (w^{**}, w^{**}) to (c, w^m) when $y < x$. Moreover, the 45^o line $y = x$ also satisfy $h(y) = h(x)$. $h(y) = h(x)$ is summarized as the blue line in Figure A-4.

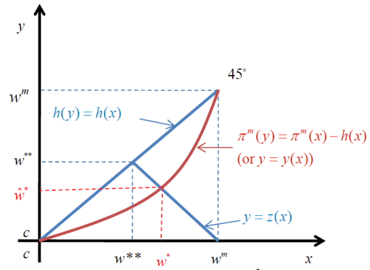


Figure A-4: Existence and Uniqueness of $\begin{cases} \pi^m(y) = \pi^m(x) - h(x) \\ h(y) = h(x) \end{cases}$

It is easy to see $\pi^m(y) = \pi^m(x) - h(x)$ and $h(y) = h(x)$ has a unique intersection point $(w^*, \widehat{w}^*)$ for $\widehat{w}^* < w^*$.

For $c \leq x < w^{**}$, Assumption 4 guarantees that $h(x) - h(y(x)) > 0 (\because h'(w) > 0, \forall w < w^{**}$ and $y(x) < x < w^{**})$.

For $w^{**} \leq x < w^*$, $h(x) - h(y(x)) > h(x) - h(z(x)) = 0 (\because h'(w) > 0, \forall w < w^{**}$ and $y(x) < z(x) < w^{**})$.

For $w^* < x < w^m$, note that $z(x) < w^{**} < x$ and $z(x) < y(x) < x$. Assumption 4 implies that, if $z(x) < y(x) \leq w^{**}$, then $h(x) - h(y(x)) < h(x) - h(z(x)) = 0$; if $w^{**} < y(x) < x$, then $h(x) - h(y(x)) < 0$.

Therefore, we have $h(x) - h(y(x)) \gtreqless 0, \forall x \lesseqgtr w^*, x \in (c, w^m)$.

Recall that $f'(x) = [\frac{-\pi^{m\prime\prime}(x)}{\pi^{m\prime}(y(x))}] \cdot [h(x) - h(y(x))]$. Thus, we have $f'(x) \gtreqless 0, \forall x \lesseqgtr w^*, x \in (c, w^m)$. That is, $f(x)$ is a well-defined single-peaked function with the peak at w^* in (c, w^m).

And $(w^*, \widehat{w}^*) \in (c, w^m) \times (c, w^m)$ is uniquely determined by $\begin{cases} h(\widehat{w}^*) = h(w^*) \\ \pi^m(\widehat{w}^*) = \pi^m(w^*) - h(w^*) \end{cases}$. ■

From the equivalence of two optimization problems, we know that the equilibrium must exist and it is uniquely determined. So Proposition 2 follows.

Appendix.2 Equilibrium Analysis: The Case of Imperfect Substitutes

The proof of Proposition 5 will follow from Lemmas 8 through 18.

■ Date 3: Retailer's Purchase Decision

Similarly, denote $w_A^e \equiv \min\{w_A, p_A(Q_o, 0)\}$. Denote $\psi(w_A^e, Q_o)$ as $p_A(Q_o, \psi) = w_A^e$. But now, the highest price manufacturer B can charge for a positive sale in (AA) is no longer w_A^e, because the two products are differentiated, and B does not need to undercut below w_A^e for a sale. Instead, it is $\overline{w}_B \equiv p_B(q_A^m(w_A^e), 0)$. In (AA), when $w_B < p_B(Q_o, \psi(w_A^e, Q_o))$, the retailer will stop buying from A after fulfilling Q_o requirement, and buy extra units from B. Then $r^{AA} = \max_{q_B}[R(Q_o, q_B) - w_B q_B] - T_o$. When $\overline{w}_B = p_B(q_A^m(w_A^e), 0) \leq w_B$, the retailer will buy exclusively from A. Then $r^{AA} = \max_{\eta \geq 0}[R(Q_o + \eta, 0) - w_A \eta] - T_o$.

Lemma 8 (Properties of Two Cutoff Lines) *(i) When $w_A < p_A(Q_o, 0)$ (AA-i hereafter), (a) both $w_B = p_B(Q_o, \psi(w_A^e, Q_o))$ and $w_B = p_B(q_A^m(w_A^e), 0)$ pass the point $(p_A(Q_o, 0), p_B(Q_o, 0))$; (b) $0 < \frac{dw_B}{dw_A}\Big|_{w_B = p_B(q_A^m(w_A^e), 0)} < 1 < \frac{dw_B}{dw_A}\Big|_{w_B = p_B(Q_o, \psi(w_A^e, Q_o))}$; (ii) When $p_A(Q_o, 0) \leq w_A$ (AA-ii hereafter), two curves coincide with $w_B = p_B(Q_o, 0)$.*

Proof of Lemma 8. (i): When $w_A < p_A(Q_o, 0)$, $w_A^e = w_A$ and $\psi(w_A^e, Q_o) > 0$. For $w_B = p_B(Q_o, \psi(w_A, Q_o))$, direct computation verifies it passes $(p_A(Q_o, 0), p_B(Q_o, 0))$. Differentiating $w_B = p_B(Q_o, \psi(w_A, Q_o))$ w.r.t. w_A, $\frac{dw_B}{dw_A} = R_{BB} \cdot \frac{\partial \psi(w_A, Q_o)}{\partial w_A} = \frac{R_{BB}}{R_{AB}} > 1$, where the second equality follows from the definition of $\psi(w_A, Q_o)$ and the inequality follows from Assumption 5. By the same vein, we can prove for $w_B = p_B(q_A^m(w_A), 0)$. (ii): When $p_A(Q_o, 0) \leq w_A$, $w_A^e = p_A(Q_o, 0)$ and $\psi(w_A^e, Q_o) = 0$. And thus $p_B(Q_o, \psi(w_A^e, Q_o)) = p_B(Q_o, 0) = p_B(q_A^m(w_A^e), 0)$. ■

The retailer's optimal response can be summarized in Figure B-1 and stated in the lemma below.

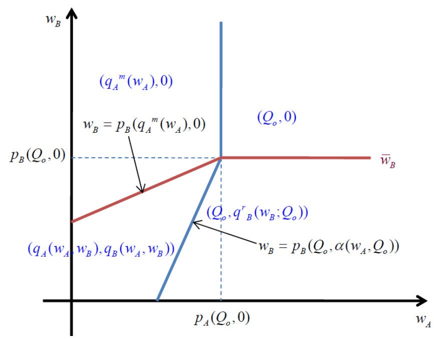


Figure B-1: The Retailer's Optimal Purchase Decision in (AA)

Lemma 9 (Retailer's Profit in (AA)) *In (AA), the retailer's profit function can be summarized as*

$$r^{AA} = \begin{cases} \max_y[R(Q_o,y) - w_B y] - T_o & \text{if } w_B \le p_B(Q_o,\psi(w_A^e,Q_o)) \\ v(w_A,w_B) + w_A Q_o - T_o & \text{if } p_B(Q_o,\psi(w_A,Q_o)) < w_B \\ & \le p_B(q_A^m(w_A),0) \text{ for (AA-i) only} \\ v(w_A^e,\infty) + w_A^e Q_o - T_o & \text{if } p_B(q_A^m(w_A^e),0) < w_B \end{cases}.$$

In (NA), the retailer will buy exclusively from B. $r^{NA}(w_B) = v(\infty, w_B)$.

Given such two options——(AA) or (NA), the retailer will choose the one giving him higher profit. The following lemma tells us the properties of the retailer's profit curves associated with these two options w.r.t. w_B.

Lemma 10 (Properties of r^{AA} and r^{NA}) *(i) Slopes: (AA):*
$\frac{\partial r^{AA}}{\partial w_B} = 1\{w_B \le p_B(Q_o,\psi(w_A^e,Q_o))\}\cdot[-q_B^r(w_B;Q_o)] + 1\{p_B(Q_o,\psi(w_A,Q_o)) < w_B \le p_B(q_A^m(w_A),0)\}\cdot[-q_B(w_A,w_B)]$;
(NA): $\frac{\partial r^{NA}}{\partial w_B} = -q_B^m(w_B)$; *(ii) Cutoffs:* $\overline{w}_B = p_B(q_A^m(w_A^e),0) \le p_B(0,0)$; *(iii) Relative steepness:* $\frac{\partial r^{NA}}{\partial w_B} \le \frac{\partial r^{AA}}{\partial w_B} \le 0, \forall w_B$.

Proof of Lemma 10. (i) follows from partial differentiation w.r.t. w_B. (ii) follows from $q_A^m(w_A^e) \ge 0$ and $R_{BA} < 0$. (iii) follows from $q_B^r(w_B;Q_o) \le q_B^m(w_B), \forall Q_o \ge 0$ and $q_B(w_A,w_B) \le q_B^m(w_B), \forall w_A$. ■

(i) tells us that r^{AA} and r^{NA} are both downward sloping before reaching a plateau. And from (ii), we know that the flat part of r^{AA} emerges before that of r^{NA}. (iii) states that r^{AA} is never steeper than r^{NA} for any given w_B. Parallel to the perfect substitutes case, the possible profit improvement can only occur when r^{AA} and r^{NA} cross once.

Lemma 11 (Retailer's Choice) *In equilibrium,* $\exists$ *a unique* $\widehat{w}_B(T_o,Q_o,w_A)$*, where* $r^{NA}(\widehat{w}_B) \gtreqless r^{AA}(\widehat{w}_B;T_o,Q_o,w_A)$ *when* $w_B \lesseqgtr \widehat{w}_B$.

■ Date 2: Manufacturer B's Problem

From Lemma 11, we know that r^{AA} must cross r^{NA} uniquely from above at $\widehat{w}_B$. There are two possible cases of crossing: (B1) $\overline{w}_B \le \widehat{w}_B$; (B2) $\widehat{w}_B < \overline{w}_B$. As we will see, because of the more cutoffs in (B2) with differentiated products than those with homogeneous products, there are three subcases in (B2), as shown in Figure B-2. And B's profits and determinations of associated $\widehat{w}_B$ are given in Table B-1. In any case, B will choose the optimal w_B^* to maximize π_B.

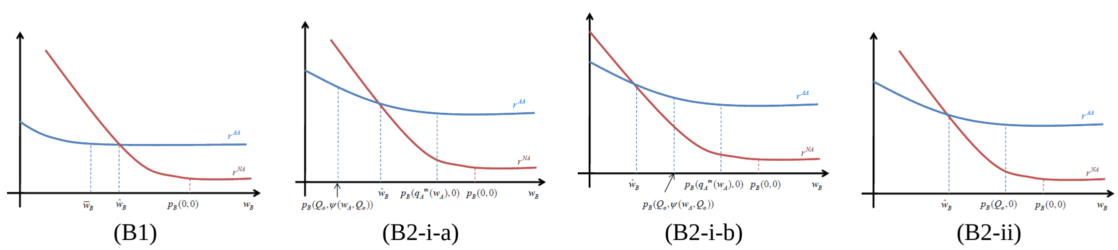


Figure B-2: Four Possible Cases of Crossings

Table B-1: Manufacturers A and B's Profits under (B1) and (B2)

	A and B's Profits and Crossing Point $\widehat{w}_B$
(B1) $\overline{w}_B \leq \widehat{w}_B$	$\pi_B = 1\{w_B \leq \widehat{w}_B\} \cdot \pi_B^m(w_B)$ (crossing point): $v(\infty, \widehat{w}_B) = v(w_A^e, \infty) + w_A^e Q_o - T_o$ $\pi_A = 1\{\widehat{w}_B < w_B^*\} \cdot [v(w_A^e, \infty) - v(\infty, \widehat{w}_B) + (w_A^e - c) q_A^m(w_A^e)]$
(B2-i-a) $p_B(Q_o, \psi(w_A, Q_o)) \leq \widehat{w}_B$ $< p_B(q_A^m(w_A), 0)$ when $w_A < p_A(Q_o, 0)$	$\pi_B = 1\{w_B \leq \widehat{w}_B\} \cdot \pi_B^m(w_B) + 1\{\widehat{w}_B < w_B \leq p_B(q_A^m(w_A), 0)\} \cdot \pi_B(w_A, w_B)$ (crossing point): $v(\infty, \widehat{w}_B) = v(w_A, \widehat{w}_B) + w_A Q_o - T_o$ $\pi_A = 1\{\widehat{w}_B < w_B^* \leq p_B(q_A^m(w_A), 0)\} \cdot [v(w_A, \widehat{w}_B) - v(\infty, \widehat{w}_B) + (w_A - c) q_A(w_A, w_B^*)]$ $+1\{p_B(q_A^m(w_A), 0) < w_B^*\} \cdot [v(w_A, \widehat{w}_B) - v(\infty, \widehat{w}_B) + (w_A - c) q_A^m(w_A)]$
(B2-i-b) $\widehat{w}_B < p_B(Q_o, \psi(w_A, Q_o))$ when $w_A < p_A(Q_o, 0)$	$\pi_B = 1\{w_B \leq \widehat{w}_B\} \cdot \pi_B^m(w_B)$ $+1\{\widehat{w}_B < w_B \leq p_B(Q_o, \psi(w_A, Q_o))\} \cdot \pi_B^r(w_B; Q_o)$ $+1\{p_B(Q_o, \psi(w_A, Q_o)) < w_B \leq p_B(q_A^m(w_A), 0)\} \cdot \pi_B(w_A, w_B)$ (crossing point): $v(\infty, \widehat{w}_B) = \max_y [R(Q_o, y) - \widehat{w}_B y] - T_o$ $\pi_A = 1\{\widehat{w}_B < w_B^* \leq p_B(Q_o, \psi(w_A, Q_o))\} \cdot \{\max_y [R(Q_o, y) - \widehat{w}_B y] - v(\infty, \widehat{w}_B) - cQ_o\}$ $+1\{p_B(Q_o, \psi(w_A, Q_o)) < w_B^* \leq p_B(q_A^m(w_A), 0)\} \cdot \{\max_y [R(Q_o, y) - \widehat{w}_B y]$ $-v(\infty, \widehat{w}_B) - w_A Q_o + (w_A - c) q_A(w_A, w_B^*)\}$ $+1\{p_B(q_A^m(w_A), 0) < w_B^*\} \cdot \{\max_y [R(Q_o, y) - \widehat{w}_B y] - v(\infty, \widehat{w}_B) - w_A Q_o + (w_A - c) q_A^m(w_A)\}$
(B2-ii) $\widehat{w}_B < p_B(Q_o, 0)$ when $p_A(Q_o, 0) \leq w_A$	$\pi_B = 1\{w_B \leq \widehat{w}_B\} \cdot \pi_B^m(w_B) + 1\{\widehat{w}_B < w_B < p_B(Q_o, 0)\} \cdot \pi_B^r(w_B; Q_o)$ (crossing point): $v(\infty, \widehat{w}_B) = \max_y [R(Q_o, y) - \widehat{w}_B y] - T_o$ $\pi_A = 1\{\widehat{w}_B < w_B^*\} \cdot \{\max_y [R(Q_o, y) - \widehat{w}_B y] - v(\infty, \widehat{w}_B) - cQ_o\}$

■ Date 1: Manufacturer A's Problem

By substituting T_o using those in Table B-1. A's profits are listed in Table B-2, too.

As shown in Lemma 12 and 13, (B1) and (B2-i-a) will be eliminated from the equilibrium. So only (B2-i-b) and (B2-ii) will emerge as the equilibrium.

Lemma 12 *In equilibrium,* $c < \widehat{w}_B < \overline{w}_B$.

Proof of Lemma 12. $\widehat{w}_B < \overline{w}_B$: Suppose not. Then it would be (B1). In this case, the only possibility for A to earn some profit is that $\widehat{w}_B < w_B^*$, but B will get 0 then. As a result, for (B1) to be the equilibrium, A has to makes $\widehat{w}_B < c$ so that $\overline{w}_B \leq \widehat{w}_B < c = w_B^*$. And $\overline{w}_B < c$ is equivalent to $p_B(q_A^m(w_A^e), 0) < c < p_B(q_A^m(c), 0)$, where the second inequality follows from Assumption 6. This implies $w_A^e < c$. Then $\pi_A = v(w_A^e, \infty) + (w_A^e - c) q_A^m(w_A^e) - v(\infty, \widehat{w}_B) < v(c, \infty) - v(\infty, \widehat{w}_B) < v(c, \infty) - v(\infty, c) < \pi_A^{2PT}$, where the first inequality follows from $w_A^e < c$, and the second one follows from $\widehat{w}_B < c$, and the last one follows from the fact that $w_A = c, T = v(c, \infty) - v(\infty, c)$ is one option in a 2PT. This contradicts with our objective of looking for profit improvement over LP or a 2PT.

$c < \overline{w}_B$: Suppose not. Then $\widehat{w}_B < \overline{w}_B \leq c = w_B^*$. Then it would be in (B2). Note $\pi_A \leq v(w_A^e, \widehat{w}_B) - v(\infty, \widehat{w}_B) + (w_A^e - c) q_A^m(w_A^e)$ for all possible cases in (B2), where the inequality for (B2-i-a) is from $w_A^e < w_A$, the one for (B2-i-b) follows from $\widehat{w}_B < p_B(Q_o, \psi(w_A, Q_o))$, and the one for (B2-ii) follows from $\widehat{w}_B < p_B(Q_o, 0)$. Recall that $\overline{w}_B \leq c$ implies $w_A^e < c$. So $\pi_A \leq v(w_A^e, \widehat{w}_B) - v(\infty, \widehat{w}_B) + (w_A^e - c) q_A^m(w_A^e) < v(w_A^e, c) - v(\infty, c) + (w_A^e - c) q_A^m(w_A^e) < v(c, c) - v(\infty, c) < \pi_A^{2PT}$, where the second inequality follows from $\frac{\partial [v(w_A^e, \widehat{w}_B) - v(\infty, \widehat{w}_B)]}{\partial \widehat{w}_B} > 0$ and $\widehat{w}_B < c$, the third one follows from $w_A^e < c$, and the last one follows from $w_A = c, T = v(c, c) - v(\infty, c)$ is one option in a 2PT. This contradicts with our objective of looking for profit improvement over LP or a 2PT.

$c < \widehat{w}_B$: Suppose not. Then $\widehat{w}_B \leq c < w_B^* < \overline{w}_B$. From Table B-1, note that in any case of (B2), π_A is increasing in $\widehat{w}_B$. Thus we can increase π_A by increasing $\widehat{w}_B$ until $\widehat{w}_B > c$ because this will not violate B's outside option by setting $w_B^* \leq \widehat{w}_B$ and being a monopoly as long as $\widehat{w}_B$ is close to c enough. Therefore, when $\widehat{w}_B \leq c$, there is always a strictly profitable deviation. ■

However, as we can see from the following lemma, (B2-i-a) is reduced to 2PT equilibrium and thus can be eliminated from our search for profitable improvement.

Lemma 13 *(B2-i-a) results in the same profit for A as that in 2PT equilibrium.*

Proof of Lemma 13. In (B2-i-a), because B would never set $p_B(q_A^m(w_A), 0) \leq w_B^*$, the only way for A to earn strictly positive profit is to make sure $\max_{w_B \leq \widehat{w}_B} \pi_B^m(w_B) \leq \max_{\widehat{w}_B < w_B < p_B(q_A^m(w_A), 0)} \pi_B(w_A, w_B)$. So by the same reasoning as the homogeneous products case, we can write A's problem as $\max_{w_A, \widehat{w}_B} v(w_A, \widehat{w}_B) - v(\infty, \widehat{w}_B) + (w_A - c) q_A(w_A, B(w_A))$ $s.t. \pi_B(w_A, B(w_A)) = \pi_B^m(\widehat{w}_B)$, which is exactly the same as 2PT one. ■

Because we are looking for a profitable improvement over LP or a 2PT, the two lemmas above tell us that we can focus on (B2-i-b) and (B2-ii) only. It turns out that they both result in the same equilibrium and can be synthesized to one case, which is summarized in Proposition 5. And from the characterization of the equilibrium, we will see it improves A's profit over LP or a 2PT.

■ Characterization of the Equilibrium

We begin with some results relating to B's profit curves, which will be used soon.

Lemma 14 *(i)* $\pi_B^m(w) \geq \max\{\pi_B(w_A, w), \pi_B^r(w; Q_o)\}, \forall w \geq c$, *with "=" at* $w = c, \forall w_A > 0, \forall Q_o > 0$.
(ii) $\pi_B^{m\prime}(w) \geq \max\{\frac{\partial \pi_B(w;w_A)}{\partial w}, \frac{\partial \pi_B^r(w;Q_o)}{\partial w}\}, \forall w \geq c, \forall w_A > 0, \forall Q_o > 0.$

Proof of Lemma 14. (i) follows from the fact that $\pi_B^m(w) = \pi_B(w_A = \infty, w) = \pi_B^r(w; Q_o = 0)$ with the substitutability between A and B. (ii) follows from Assumption 7. ■

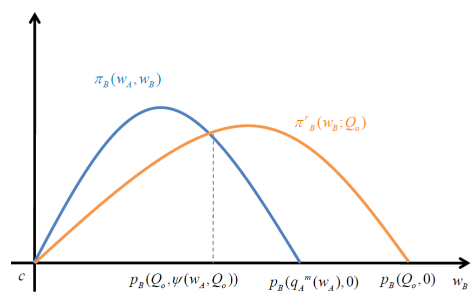
The next lemma tells us the properties of two curves—$\pi_B^r(w_B; Q_o)$ and $\pi_B(w_A, w_B)$.

Lemma 15 *(i) When* $c \leq p_B(Q_o, \psi(w_A, Q_o))$, $\pi_B^r(w_B; Q_o) \lesseqgtr \pi_B(w_A, w_B)$ *when* $w_B \lesseqgtr p_B(Q_o, \psi(w_A, Q_o))$. *(ii)* $\frac{\partial \pi_B(w_A,w_B)}{\partial w_B}\Big|_{w_B = p_B(Q_o,\psi(w_A,Q_o))} < \frac{\partial \pi_B^r(w_B;Q_o)}{\partial w_B}\Big|_{w_B = p_B(Q_o,\psi(w_A,Q_o))}$.

Proof of Lemma 15. (i): At $w_B = p_B(Q_o, \psi(w_A, Q_o))$, $q_B^r(w_B; Q_o) = \psi(w_A, Q_o) = q_B(w_A, w_B)$, $q_A(w_A, w_B) = Q_o$. When $w_B \lesseqgtr p_B(Q_o, \psi(w_A, Q_o))$, then $q_A(w_A, w_B) \lesseqgtr Q_o$. And $p_B(Q_o, q_B^r(w_B; Q_o)) = w_B = p_B(q_A(w_A, w_B), q_B(w_A, w_B))$. Hence, when $w_B \lesseqgtr p_B(Q_o, \psi(w_A, Q_o))$, $q_B^r(w_B; Q_o) \lesseqgtr q_B(w_A, w_B)$. Remember that $\pi_B^r(w_B; Q_o) = (w_B - c) q_B^r(w_B; Q_o)$ and $\pi_B(w_A, w_B) = (w_B - c) q_B(w_A, w_B)$, so this part follows.

(ii): $[\frac{\partial \pi_B(w_A,w_B)}{\partial w_B} - \frac{\partial \pi_B^r(w_B;Q_o)}{\partial w_B}]\Big|_{w_B = p_B(Q_o,\psi(w_A,Q_o))} = (w_B - c) \cdot \frac{R_{AB}^2}{(R_{AA}R_{BB} - R_{AB}^2)R_{BB}}\Big|_{(Q_o,\psi(w_A,Q_o))} < 0.$ ■

This lemma shows that when $c \leq p_B(Q_o, \psi(w_A, Q_o))$, $\pi_B^r(w_B; Q_o)$ crosses $\pi_B(w_A, w_B)$ from below at $w_B = p_B(Q_o, \psi(w_A, Q_o))$. They are as shown in Figure B-3.



Fgiure B-3: $\pi_B^r(w_B; Q_0)$ and $\pi_B(w_A, w_B)$

■ (B2-i-b) $w_A < p_A(Q_o, 0)$ and $\widehat{w}_B < p_B(Q_o, \psi(w_A, Q_o))$ (so $c < p_B(Q_o, \psi(w_A, Q_o))$)

In this subcase, B's profit is depicted as the red line in Figure B-4 below.

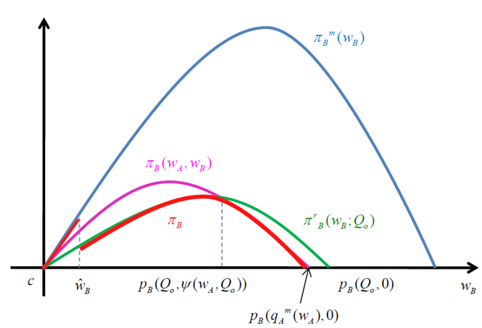


Figure B-4: Manufacturer B's Profit Curve in (B2-i-b)

Lemma 16 *In equilibrium,* $\widehat{w}_B < w_B^* < p_B(Q_o, \psi(w_A, Q_o))$, *and thus* $\max_{x \leq \widehat{w}_B} \pi_B^m(x) \leq \pi_B^r(w_B^*; Q_o)$.

Proof of Lemma 16. Suppose not. Then $\pi_A^{3PT} < v(w_A, \widehat{w}_B) - v(\infty, \widehat{w}_B) + \pi_A(w_A, B(w_A)) < \pi_A^{2PT}$, where the first inequality follows from $\pi_A(w_A, w_B^*) < \pi_A(w_A, B(w_A))$ and $w_A < p_A(Q_o, 0)$. ■

Denote program [P*] as

$$\text{[P*]} \qquad (Q_o^*, w_B^*, \widehat{w}_B^*) = \arg \max_{\substack{(q,x,y) \\ y<x<w_B^m}} \left\{ \begin{array}{c} \max_z [R(q,z) - y \cdot z] - v(\infty, y) - c \cdot q \\ s.t. \pi_B^m(y) = \pi_B^r(x;q) \\ \frac{\partial \pi_B^r(x;q)}{\partial x} = 0 \end{array} \right\}.$$

Lemma 17 *In (B2-i-b), (i) $\pi_B^{m\prime}(\widehat{w}_B) > 0$. (ii) $\widehat{w}_B < w_B^* < p_B(Q_o, \psi(w_A, Q_o))$ and $\frac{\partial \pi_B^r(w_B^*; Q_o)}{\partial w_B} = 0$;*
(iii) $\pi_B^r(w_B^*; Q_o) = \pi_B^m(\widehat{w}_B) = \max_{x \leq \widehat{w}_B} \pi_B^m(x)$. The 3PT equilibrium $(T_o^*, Q_o^*, w_A^*; w_B^*)$ is characterized by

$$T_o^* = \max_y [R(Q_o^*, y) - \widehat{w}_B^* y] - v(\infty, \widehat{w}_B^*) \qquad \textit{(B.1)}$$

$$p_A(Q_o^*, q_B^r(w_B^*; Q_o)) \leq w_A^* < p_A(Q_o^*, 0), \qquad \textit{(B.2)}$$

where $(Q_o^*, w_B^*, \widehat{w}_B^*)$ is the solution to the program [P].*

In this equilibrium, the leader earns higher profit than that under LP or a 2PT.

Proof of Lemma 17. (i): Suppose not. That is, $\pi_B^{m\prime}(\widehat{w}_B) \leq 0$. From Lemma 14, we know that $\frac{\partial \pi_B^r(\widehat{w}_B; Q_o)}{\partial w_B} \leq \pi_B^{m\prime}(\widehat{w}_B) \leq 0$, which implies $\max_{\widehat{w}_B < x < p_B(Q_o, \psi(w_A, Q_o))} \pi_B^r(x; Q_o) = \pi_B^r(\widehat{w}_B; Q_o)$. However, $\pi_B^r(\widehat{w}_B; Q_o) < \pi_B^m(\widehat{w}_B)$, $\forall \widehat{w}_B > c$ from Lemma 14, which contradicts with $\max_{x \leq \widehat{w}_B} \pi_B^m(x) \leq \pi_B^r(w_B^*; Q_o)$.

(ii): Because $\pi_B^r(\widehat{w}_B; Q_o) < \pi_B^m(\widehat{w}_B)$ from Lemma 14, in order for $\pi_B^r(w_B^*; Q_o) \geq \pi_B^m(\widehat{w}_B)$ to hold, we must have $\frac{\partial \pi_B^r(\widehat{w}_B; Q_o)}{\partial w_B} > 0$ and $\widehat{w}_B < w_B^*$. Because w_A doesn't enter the objective function, we can increase it such that $\frac{\partial \pi_B^r(w_B; Q_o)}{\partial w_B} \Big|_{w_B = p_B(Q_o, \psi(w_A, Q_o))} \leq 0$.

(iii): Suppose not. Notice that $\frac{\partial \pi_A}{\partial \widehat{w}_B} = -q_B^r(\widehat{w}_B; Q_o) + q_B^m(\widehat{w}_B) > 0$, then we can increase π_A by increasing $\widehat{w}_B$ until $\pi_B^r(w_B^*; Q_o) \geq \pi_B^m(\widehat{w}_B)$ becomes binding.

By the definition of $\psi(w_A, Q_o)$, $w_B \leq p_B(Q_o, \psi(w_A, Q_o)) \iff p_A(Q_o, q_B^r(w_B; Q_o)) \leq w_A$. ■

■ (B2-ii) $p_A(Q_o, 0) \leq w_A$ and $\widehat{w}_B < p_B(Q_o, 0)$

In this case, $\overline{w}_B = p_B(Q_o, 0)$. From Lemma 12 $c < \widehat{w}_B < \overline{w}_B$, we know that B's profit is as the red line in Figure B-5 below.

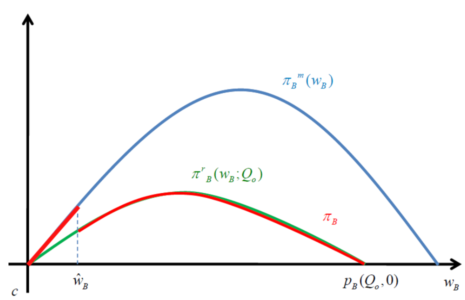


Figure B-5: Manufacturer B's Profit Curve in (B2-ii)

Thus, for possible positive profit, A must ensure $\widehat{w}_B < w_B^* < p_B(Q_o, 0)$. This is equivalent to $0 \leq \max_{x \leq \widehat{w}_B} \pi_B^m(x) \leq \pi_B^r(w_B^*; Q_o)$, where the second inequality comes from $c < \widehat{w}_B$. Parallel, we can characterize the equilibrium of case (B2-ii) as follows.

Lemma 18 *In (B2-ii), 3PT equilibrium $(T_o^*, Q_o^*, w_A^*; w_B^*)$ is characterized by*

$$T_o^* = \max_y [R(Q_o^*, y) - \widehat{w}_B^* y] - v(\infty, \widehat{w}_B^*) \qquad \textit{(B.3)}$$

$$p_A(Q_o^*, 0) \leq w_A^*, \qquad \textit{(B.4)}$$

where $(Q_o^*, w_B^*, \widehat{w}_B^*)$ is the solution to the program [P].*

In this equilibrium, the leader earns higher profit than that under LP or a 2PT.

Compared with (B2-i-b), the only difference here is constraint (B.4) vs. constraint (B.2). Note that w_A does not enter either the objective function, nor any of the constraints. Therefore, two equilibria are the same. Since $(q,x,y) \in [0,q_A^m] \times [c,w_B^m] \times [c,w_B^m]$, the equilibrium exists by the Weierstrass Theorem. So Proposition 5 follows.

■ Proof of Proposition 6

The proof of Proposition 6 will follow from Lemmas 19 through 24.

First, it is easy to see $\pi_A^{LP} < \pi_A^{2PT} < \pi_A^{3PT}$ since more complex pricing scheme should give A higher profit.

We first compare 2PT and LP equilibrium.

Letting ξ denote the Lagrange multiplier for 2PT's program, the first-order conditions[29] are

$$(w): \frac{d\pi_A(w,B(w))}{dw} - q_A(w,\widehat{w}) + \xi \cdot \frac{\partial \pi_B(w,B(w))}{\partial w_A} = 0$$
$$(\widehat{w}): -q_B(w,\widehat{w}) + q_B^m(\widehat{w}) - \xi \cdot \pi_B^{m'}(\widehat{w}) = 0$$
$$(\xi): \pi_B^m(\widehat{w}) = \pi_B(w,B(w))$$

Eliminating ξ and denoting the solution to $\pi_B^m(\widehat{w}) = \pi_B(w,B(w))$ as $\widehat{w} = k(w)$, we know w^{2PT} is characterized by

(B.7) $$\frac{d\pi_A(w,B(w))}{dw} = q_A(w,k(w)) - [q_B^m(k(w)) - q_B(w,k(w))] \cdot \frac{\frac{\partial \pi_B(w,B(w))}{\partial w_A}}{\pi_B^{m'}(k(w))},$$

and $k'(w) = \frac{\frac{\partial \pi_B(w,B(w))}{\partial w_A}}{\pi_B^{m'}(k(w))} > 0$. Note that w_A^{LP} is characterized by $\frac{d\pi_A(w_A^{LP},B(w_A^{LP}))}{dw} = 0$. So the sign of (B.7)'s RHS above determines whether $w_A^{2PT} < w_A^{LP}$ or not.

Lemma 19 $\frac{\partial \pi_B(w,B(w))}{\partial w_A} < \pi_B^{m\prime}(k(w)), \forall w.$

Proof of Lemma 19. Let $j(w) \equiv \pi_B^{m\prime}(k(w)) - \frac{\partial \pi_B(w,B(w))}{\partial w_A}$.

$j'(w) = \pi_B^{m\prime\prime}(k(w))k'(w) - [\frac{\partial^2 \pi_B(w,B(w))}{\partial w_A^2} - \frac{\partial^2 \pi_B(w,B(w))}{\partial w_A \partial w_B} \cdot \frac{\frac{\partial^2 \pi_B(w,B(w))}{\partial w_A \partial w_B}}{\frac{\partial^2 \pi_B(w,B(w))}{\partial w_B^2}}] < 0$, where the inequality follows from parts (i) and (ii) of Assumption 7. Since $j(\infty) = 0 (\because k(w) = B(\infty) = w_B^m), j(w) > 0, \forall w.$ ■

Lemma 20 $q_B^m(w) < q_A(w,\widehat{w}) + q_B(w,\widehat{w}), \forall w, \forall \widehat{w}.$

Proof of Lemma 20. For $q_B(t,\widehat{w})$ s.t. $R_B(t,q_B) = \widehat{w}$, $\frac{\partial q_B(t,\widehat{w})}{\partial t} = -\frac{R_{AB}}{R_{BB}} \in (-1,0)$.

$q_B^m(w) - q_B(w,\widehat{w}) = \int_{q_A(w,\widehat{w})}^0 \frac{\partial q_B(t,\widehat{w})}{\partial t} dt = \int_0^{q_A(w,\widehat{w})} [-\frac{\partial q_B(t,\widehat{w})}{\partial t}] dt < \int_0^{q_A(w,\widehat{w})} 1 dt = q_A(w,\widehat{w}).$ ■

These two lemmas help us determine the sign of (B.7)'s RHS.

$$\begin{aligned} \frac{d\pi_A(w,B(w))}{dw} &= q_A(w,k(w)) - [q_B^m(k(w)) - q_B(w,k(w))] \cdot \frac{\frac{\partial \pi_B(w,B(w))}{\partial w_A}}{\pi_B^{m'}(k(w))} \\ &> q_A(w,k(w)) - [q_B^m(k(w)) - q_B(w,k(w))] \cdot 1 \\ &> 0. \end{aligned}$$

Hence, from the concavity of $\pi_A(w,B(w))$, we must have $w_A^{2PT} < w_A^{LP}$. So $w_B^{2PT} = B(w_A^{2PT}) < B(w_A^{LP}) = w_B^{LP}$. Moreover, $\pi_B^{2PT} = \pi_B(w_A^{2PT}, B(w_A^{2PT})) < \pi_B(w_A^{LP}, B(w_A^{LP})) = \pi_B^{LP}$ follows from the fact that $\frac{d\pi_B(w,B(w))}{dw} = \frac{\partial \pi_B(w,B(w))}{\partial w_A} > 0$. In addition, since per-unit prices for both products under a 2PT are lower than those under LP, $TS^{LP} < TS^{2PT}$.

The following lemma summarizes the comparison between 2PT equilibrium and LP equilibrium.

Lemma 21 *(i)* $w_A^{2PT} < w_A^{LP}, w_B^{2PT} < w_B^{LP}$. *(ii)* $\pi_B^{2PT} < \pi_B^{LP}$. *(iii)* $TS^{LP} < TS^{2PT}$.

Now we compare 3PT and 2PT equilibrium.

[29] The sufficiency of these conditions can be shown in a similar pattern as we show the 3PT equilibrium.

Denote $\widetilde{w} \equiv R_A(Q_o^*, q_B^r(\widehat{w}; Q_o^*))$. Then $Q_o^* = q_A(\widetilde{w}, \widehat{w}), q_B^r(\widehat{w}; Q_o^*) = q_B(\widetilde{w}, \widehat{w})$. Then the objective function of 3PT's program [P*] can be written as $\max_z [R(Q_o^*, z) - \widehat{w} \cdot z] - v(\infty, \widehat{w}) - c \cdot Q_o^* = \pi_A(\widetilde{w}, \widehat{w}) + v(\widetilde{w}, \widehat{w}) - v(\infty, \widehat{w})$. So instead of choosing $(Q_o, w^*, \widehat{w})$, it is equivalent to choose $(\widetilde{w}, \widehat{w})$ to solve

$$\max_{\widetilde{w}, \widehat{w}} \pi_A(\widetilde{w}, \widehat{w}) + v(\widetilde{w}, \widehat{w}) - v(\infty, \widehat{w})$$
$$s.t. \pi_B^m(\widehat{w}) = \max_x \pi_B^r(x; q_A(\widetilde{w}, \widehat{w})).$$

Lemma 22 $q_B(\widetilde{w}, x) < q_B^r(x; q_A(\widetilde{w}, \widehat{w})), \pi_B(\widetilde{w}, x) < \pi_B^r(x; q_A(\widetilde{w}, \widehat{w})), \forall \widehat{w} < x.$

Proof of Lemma 22. $\forall \widehat{w} < x$, we have $q_A(\widetilde{w}, \widehat{w}) < q_A(\widetilde{w}, x)$, which follows from $\frac{\partial q_A}{\partial w_B} > 0$. By definitions of $q_B^r(x; q_A(\widetilde{w}, \widehat{w}))$ and $q_B(\widetilde{w}, x)$, $R_B(q_A(\widetilde{w}, \widehat{w}), q_B^r(x; q_A(\widetilde{w}, \widehat{w}))) = x = R_B(q_A(\widetilde{w}, x), q_B(\widetilde{w}, x))$. So $q_B(\widetilde{w}, x) < q_B^r(x; q_A(\widetilde{w}, \widehat{w}))$, and thus $\pi_B(\widetilde{w}, x) = (x - c) q_B(\widetilde{w}, x) < (x - c) q_B^r(x; q_A(\widetilde{w}, \widehat{w})) = \pi_B^r(x; q_A(\widetilde{w}, \widehat{w}))$. ■

Therefore, if we substitute 2PT equilibrium prices $(w_A^{2PT}, \widehat{w}^{2PT})$ into 3PT's constraint, then

$$\begin{aligned} \pi_B^m(\widehat{w}^{2PT}) &= \pi_B(w_A^{2PT}, B(w_A^{2PT})) \\ &< \pi_B^r(B(w_A^{2PT}); q_A(w_A^{2PT}, \widehat{w}^{2PT})) \text{ (using Lemma 22)} \\ &\leq \max_x \pi_B^r(x; q_A(w_A^{2PT}, \widehat{w}^{2PT})). \end{aligned}$$

Hence, under 2PT equilibrium prices $(w_A^{2PT}, \widehat{w}^{2PT})$, 3PT's constraint is not binding. But 3PT's objective function is increasing in $\widehat{w}$ when constraint is not binding. So we must have $\widehat{w}^{2PT} < \widehat{w}^{3PT}$.

By definition of $\underline{w}_A$,

$$\begin{aligned} \pi_A^{3PT} &= \pi_A(\widetilde{w}^{3PT}, \widehat{w}^{3PT}) + v(\widetilde{w}^{3PT}, \widehat{w}^{3PT}) - v(\infty, \widehat{w}^{3PT}) \\ &= \pi_A(\underline{w}_A, w_B^*) + (\widetilde{w}^{3PT} - \underline{w}_A) \cdot Q_o^* + v(\widetilde{w}^{3PT}, \widehat{w}^{3PT}) - v(\infty, \widehat{w}^{3PT}) \\ \pi_B^{3PT} &= \pi_B^r(w_B^*; Q_o^*) = \pi_B(\underline{w}_A, w_B^*) \\ \pi_R^{3PT} &= v(\underline{w}_A, w_B^*) - (\widetilde{w}^{3PT} - \underline{w}_A) \cdot Q_o^* - [v(\widetilde{w}^{3PT}, \widehat{w}^{3PT}) - v(\infty, \widehat{w}^{3PT})]. \end{aligned}$$

So for welfare comparison, the "effective per-unit price" for A under a 3PT is $\underline{w}_A$ and per-unit price for B is w_B^*.

Lemma 23 *(i)* $\pi_B^{2PT} < \pi_B^{3PT}$. *(ii)* $w_A^{2PT} < \underline{w}_A$. *(iii)* $w_B^{2PT} < B(\underline{w}_A) < w_B^*$.

Proof of Lemma 23. (i) $\because \widehat{w}^{2PT} < \widehat{w}^{3PT} \therefore \pi_B^{2PT} = \pi_B^m(\widehat{w}^{2PT}) < \pi_B^m(\widehat{w}^{3PT}) = \pi_B^{3PT}$.

(ii) $\pi_B^{3PT} = (w_B^* - c) q_B^r(w_B^*; Q_o) = (w_B^* - c) q_B(\underline{w}_A, w_B^*) < \max_x \pi_B(\underline{w}_A, x) = \pi_B(\underline{w}_A, B(\underline{w}_A))$. Combining with (i), we have $\pi_B(\underline{w}_A, B(\underline{w}_A)) > \pi_B^{3PT} > \pi_B^{2PT} = \pi_B(w_A^{2PT}, B(w_A^{2PT}))$. So $w_A^{2PT} < \underline{w}_A$ follows from the fact that $\frac{d\pi_B(w, B(w))}{dw} = \frac{\partial \pi_B(w, B(w))}{\partial w_A} > 0$.

(iii) $\because q_B^r(w_B^*; Q_o^*) = q_B(\underline{w}_A, w_B^*) \therefore \pi_B^r(w_B^*; Q_o^*) = \pi_B(\underline{w}_A, w_B^*)$. From part (ii) of Lemma 15, $\frac{\partial \pi_B(\underline{w}_A, w_B^*)}{\partial w_B} < 0$. So $w_B^* > B(\underline{w}_A)$. And from (ii), $B(\underline{w}_A) > B(w_A^{2PT}) = w_B^{2PT}$. ■

So the "effective per-unit price" for both A and B under a 3PT are higher than those under a 2PT. We can conclude that $TS^{3PT} < TS^{2PT}$. Combining with the fact that $\pi_A^{3PT} > \pi_A^{2PT}$ and $\pi_B^{3PT} > \pi_B^{2PT}$, we must have $\pi_R^{3PT} < \pi_R^{2PT}$. Consequently, we can summarizes the comparison between 3PT equilibrium and 2PT equilibrium as below.

Lemma 24 *(i)* $w_A^{2PT} < \underline{w}_A^{3PT}, w_B^{2PT} < w_B^*$. *(ii)* $\pi_B^{2PT} < \pi_B^{3PT}$. *(iii)* $TS^{3PT} < TS^{2PT}$. *(iv)* $\pi_R^{3PT} < \pi_R^{2PT}$.

Combining Lemmas 21 and 24, Proposition 6 follows.

■ Proof of Proposition 7

It is easy to verify Assumptions 5~7 are satisfied in general differentiated linear demands. So the existence and characterization of the equilibrium is guaranteed by Proposition 5. Here we will focus on the uniqueness of the equilibrium.

In general differentiated linear demands, from the constraint $\frac{\partial \pi_B^r(x;q)}{\partial x} = 0$, we have $q = l(x) \equiv \frac{\alpha + c - 2x}{\beta}$. Then from the constraint $\pi_B^m(y) = \pi_B^r(x; q)$ with $q = \frac{\alpha + c - 2x}{\beta}$, we can solve for $y = g(x) \equiv \frac{\alpha + c - \sqrt{(\alpha - c)^2 - 4(x - c)^2}}{2} < x$. Thus, the program [P*] now becomes a single-variable optimization as below, after eliminating q and y with substitution.

$$[\widetilde{P}^*] \qquad \max_{x < w_B^m} \max_z [R(l(x), z) - g(x) \cdot z] - v(\infty, g(x)) - c \cdot l(x)$$

Denote the objective function in the program $[\widetilde{P}^*]$ as $F(x) \equiv \max_z [R(l(x), z) - g(x) \cdot z] - v(\infty, g(x)) - c \cdot l(x)$. Next, we will establish the uniqueness of its solution by showing that $F(x)$ is a well-defined single-peaked function with the peak at $w_B^* \in (c, w_B^m)$.

From the continuity and differentiability of $R(\cdot,\cdot)$, $l(\cdot)$, $g(\cdot)$ and $v(\cdot,\cdot)$, we can see that $F(x)$ is continuous and differentiable, $\forall x \in [c, w_B^m]$.

$$\begin{aligned} F'(x) &= [R_A(l(x), q_B^r(g(x); l(x))) - c] \cdot l'(x) + [q_B^m(g(x)) - q_B^r(g(x); l(x))] \cdot g'(x) \\ &= \frac{-l'(x)}{\pi_B^{m\prime}(g(x))} \cdot \{[q_B^m(g(x)) - q_B^r(g(x); l(x))] \cdot (-\frac{\partial \pi_B^r(x; l(x))}{\partial q}) - [R_A(l(x), q_B^r(g(x); l(x))) - c] \cdot \pi_B^{m\prime}(g(x))\}. \end{aligned}$$

Note that $\frac{-l'(x)}{\pi_B^{m\prime}(g(x))} > 0$. Let $H(x, y) \equiv [q_B^m(y) - q_B^r(y; l(x))] \cdot (-\frac{\partial \pi_B^r(x; l(x))}{\partial q}) - [R_A(l(x), y) - c] \cdot \pi_B^{m\prime}(y)$. Thus,

$$\text{the sign of } F'(x) = \text{the sign of } H(x, g(x)).$$

If we denote $\gamma \equiv \alpha - c, \widetilde{x} \equiv x - c, \widetilde{y} \equiv y - c$, then

$$H(x, y) = \beta \cdot \widetilde{x} \cdot (\gamma - 2\widetilde{x}) - \beta \cdot \widetilde{y} \cdot (\gamma - 2\widetilde{y}) - (\gamma - \beta \cdot 2\widetilde{x} - \frac{\gamma - 2\widetilde{x}}{\beta}) \cdot (\gamma - 2\widetilde{y}),$$

which is a hyperbolic paraboloid.

First, analytical geometry tells us $H(x, y) = 0$ is a hyperbola with center $(x_o, y_o) = (\frac{2(1-\beta^2-\frac{\beta}{2})^2+\frac{\beta^2}{2}}{4[(1-\beta^2)^2+\beta^4]} \cdot \gamma, \frac{2(1-\beta^2)^2+\beta^2(1-\beta)^2+\beta^4}{4[(1-\beta^2)^2+\beta^4]} \cdot \gamma)$ and two asymptotes $\frac{y-y_o}{x-x_o} = \frac{\sqrt{\beta^4+(1-\beta^2)^2} \pm (1-\beta^2)}{\beta^2}$.

Note that $\beta \in [0, 1]$. It is easy to verify that its center (x_o, y_o) lies above 45^o line, and the slopes of two asymptotes are $\frac{\sqrt{\beta^4+(1-\beta^2)^2}-(1-\beta^2)}{\beta^2} \in [0, 1]$ and $-\frac{\sqrt{\beta^4+(1-\beta^2)^2}+(1-\beta^2)}{\beta^2} \in [-1, 0]$. Moreover, $H(w_B^m, w_B^m) = H(\frac{\gamma}{2}, \frac{\gamma}{2}) = 0$, $H(\frac{\gamma}{2} \cdot \frac{1}{1+\beta}, \frac{\gamma}{2} \cdot \frac{1}{1+\beta}) = 0$, and $H(x_c, 0) = 0$, where $x_c = \frac{\gamma}{2} \cdot \frac{\sqrt{(3\beta-\frac{2}{\beta})^2+8\beta(\frac{1}{\beta}-1)}+3\beta-\frac{2}{\beta}}{2\beta} + c \leq w_B^m = \frac{\gamma}{2}$.

Meanwhile, it is easy to plot $y = g(x) = \frac{\alpha+c-\sqrt{(\alpha-c)^2-4(x-c)^2}}{2}, \forall x \in [c, \frac{\gamma}{2}]$, which passes (c, c) and $(w_B^m, w_B^m) = (\frac{\gamma}{2}, \frac{\gamma}{2})$ and it is increasing in the range.

Thus, $H(x, y) = 0$ and $y = g(x)$ for general differentiated linear demands must be as show in Figure B-6 below.

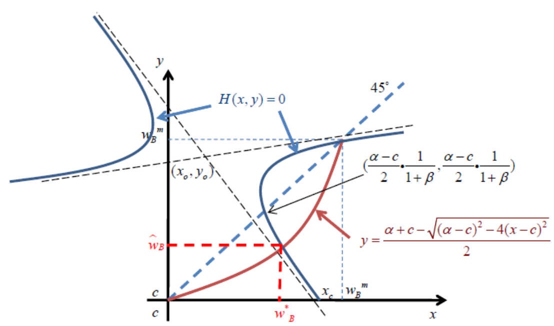


Figure B-6: $H(x, y) = 0$ and $y = g(x)$ for general differentiated linear demands

Obviously, $H(x, y) = 0$ and $y = g(x)$ must cross uniquely at $x = w_B^* \in (c, w_B^m)$ for $x \in (c, w_B^m)$.

From the property of the hyperbolic paraboloid $H(x, y)$, we have $H(x, g(x)) \gtreqless 0, \forall x \lesseqgtr w_B^*, x \in (c, w_B^m)$. This implies that $F'(x) \gtreqless 0, \forall x \lesseqgtr w_B^*, x \in (c, w_B^m)$. So $F(x)$ is a well-defined singled-peaked function with the peak at $w_B^* \in (c, w_B^m)$. And $w_B^* \in (c, w_B^m)$ is uniquely determined by $H(w_B^*, g(w_B^*)) = 0$.

Appendix.3: Equilibrium for Extension: 2PT vs. 2PT and 3PT vs. 3PT

Proof of Proposition 12. **2PT vs. 2PT:** Note that B can at least always mimick A's offer. So in equilibrium, B must have positive sale and A's quantity sale must either be 0 or $q_A(w_A, w_B)$. Suppose $\Pi_A^{2PT} = F_A + (w_A - c) \cdot q_A(w_A, w_B) > v(c, c) - v(\infty, c) \geq 0$. We will show that B will always undercut. Given a 2PT (w_A, F_A) from A, undercut: B will ensure $v(\infty, w_B) - F_B \geq v(w_A, \infty) - F_A$. Therefore,

$$\Pi_B^{2PT} = F_B + (w_B - c) \cdot q_B^m(w_B) \leq v(\infty, w_B) + (w_B - c) \cdot q_B^m(w_B) - v(w_A, \infty) + F_A \leq v(\infty, c) - v(w_A, \infty) + F_A.$$

So the most profitable undercut will result in $v(\infty, c) - v(w_A, \infty) + F_A$. This is achievable via $\begin{cases} w_B = c \\ F_B = v(\infty, c) - v(w_A, \infty) + F_A \end{cases}$.

accommodate: B will ensure $v(w_A, w_B) - F_A - F_B \geq v(w_A, \infty) - F_A$. Therefore,

$$\begin{aligned} \Pi_B^{2PT} &= F_B + (w_B - c) \cdot q_B(w_A, w_B) \\ &\leq v(w_A, w_B) + (w_B - c) \cdot q_B(w_A, w_B) - v(w_A, \infty) \\ &< v(w_A, w_B) + (w_A - c) \cdot q_A(w_A, w_B) + (w_B - c) \cdot q_B(w_A, w_B) + F_A - v(c, c) + v(\infty, c) - v(w_A, \infty) \\ (\quad \because \quad & \text{supposition } \Pi_A^{2PT} = F_A + (w_A - c) \cdot q_A(w_A, w_B) > v(c, c) - v(\infty, c)) \qquad \text{(C.1)} \\ &\leq v(\infty, c) - v(w_A, \infty) + F_A \text{ (B's profit under most profitable undercut).} \\ (\quad \because \quad & v(w_A, w_B) + (w_A - c) \cdot q_A(w_A, w_B) + (w_B - c) \cdot q_B(w_A, w_B) \leq v(c, c)) \end{aligned}$$

Consequently, $\Pi_A^{2PT} > v(c, c) - v(\infty, c)$ is impossible.

Suppose $\Pi_A^{2PT} = F_A + (w_A - c) \cdot q_A(w_A, w_B) < v(c, c) - v(\infty, c)$. We will show that A can always increase its profit until the inequality becomes binding. This can seen that (C.1) will be reversed. That is, by setting $w_A = c$, B will be induced to set $w_B = c$ and achieve its most profit under accommodation

$$\begin{aligned} \Pi_B^{2PT} &= v(w_A, w_B) + (w_B - c) \cdot q_B(w_A, w_B) - v(w_A, \infty) \\ &> v(w_A, w_B) + (w - c) \cdot q_A(w_A, w_B) + (w_B - c) \cdot q_B(w_A, w_B) + F_A - v(c, c) + v(\infty, c) - v(w_A, \infty) \\ &\geq v(\infty, c) - v(w_A, \infty) + F_A \text{ (B's profit under most profitable undercut).} \end{aligned}$$

So A can always increase its profit by setting $w_A = c$ and increasing F_A without worrying about firm B's undercut, as long as $\Pi_A^{2PT} < v(c, c) - v(\infty, c)$. Consequently, $\Pi_A^{2PT} < v(c, c) - v(\infty, c)$ is impossible, neither.

As a result, the only equilibrium outcome will be $\Pi_A^{2PT} = v(c, c) - v(\infty, c)$. And from the proof above, B will accommodate and earn $\Pi_B^{2PT} = v(c, c) - v(c, \infty)$. Then retailer's profit follows straightforward.

3PT vs. 3PT: The proof arguments for A's profit under 3PT vs. 3PT are parallel. The only difference is that the optimal 3PT, which results in the same profit for A, is not uniquely determined. Correspondingly, B's profit depends on the threat point from the coalition of R and A. The most favorable 3PT for B can be a reduced 3PT as a price-quantity offer $(q_A(c, c), v(c, c) - v(\infty, c))$ from A. The associated threat point payoff is $R(q_A(c, c), 0) - c \cdot q_A(c, c)$. The least favorable 3PT for B can be reduced 3PT as a 2PT with $Q_A = 0, T_A = v(c, c) - v(\infty, c), w_A = c$, then the threat point payoff is $v(c, \infty)$. ■

Appendix.4: Equilibrium for General Differentiated Linear Demands

Table 1: Equillibrium Tariffs for General Differentiated Linear Demands

	LP	2PT	3PT
Fixed Fee	N/A	$T^{2PT} = \lambda^2 \cdot \frac{1}{8(1-\beta^2)^2} \cdot \{[8 - (2+\beta^2)^2]\Gamma^2 + 2\beta[(3\beta^2 - 2) - 2(1-\beta^2)\Delta]\Gamma + \beta^2[1 - 2\beta^2 + 2(1-\beta^2)\Delta]\}$	$T^{3PT} = \lambda \cdot \Phi \cdot \{c + \lambda \cdot \frac{2-[\beta(1+\Omega)+(1-\beta^2)\Phi]}{2}\}$
Quantity Threshold	N/A	N/A	$Q^{3PT} = \lambda \cdot \Phi$
Per-Unit Price w_A	$w_A^{LP} = c + \lambda \cdot \frac{(2+\beta)(1-\beta)}{2(2-\beta^2)}$	$w_A^{2PT} = c + \lambda(1 - \Gamma)$	$w_A^{3PT} \geq c + \lambda \cdot \frac{2-\beta-(2-\beta^2)\Phi}{2}$
Per-Unit Price w_B	$w_B^{LP} = c + \lambda \cdot \frac{(4+2\beta-\beta^2)(1-\beta)}{4(2-\beta^2)}$	$w_B^{2PT} = c + \lambda \cdot \frac{1-\beta\cdot\Gamma}{2}$	$w_B^{3PT} = c + \lambda \cdot \frac{1-\beta\cdot\Phi}{2}$

Table 2: Equillibrium Surpluses for General Differentiated Linear Demands

	LP	2PT	3PT
A's Profit	$\pi_A^{LP} = \lambda^2 \cdot \frac{(1-\beta)(2+\beta)^2}{8(1+\beta)(2-\beta^2)}$	$\pi_A^{2PT} = \lambda^2 \cdot \frac{1}{8(1-\beta^2)^2} \cdot \{(-4+8\beta^2-5\beta^4)\Gamma^2 + 2[4-6\beta^2+\beta^3+2\beta^4-2\beta(1-\beta^2)\Delta]\Gamma + 2\beta^2(1-\beta^2)\Delta - \beta(4-\beta-4\beta^2+2\beta^3)\}$	$\pi_A^{3PT} = \lambda^2 \cdot \frac{\Phi[2-(1-\beta^2)\Phi-\beta(1+\Omega)]}{2}$
B's Profit	$\pi_B^{LP} = \lambda^2 \cdot \frac{(1-\beta)(4+2\beta-\beta^2)^2}{16(1+\beta)(2-\beta^2)^2}$	$\pi_B^{2PT} = \lambda^2 \cdot \frac{(1-\beta\cdot\Gamma)^2}{4(1-\beta^2)}$	$\pi_B^{3PT} = \lambda^2 \cdot \frac{[1-\beta\cdot\Phi]^2}{4}$
R's Profit	$\pi_R^{LP} = \lambda^2 \cdot \frac{16(1+\beta)(2-\beta^2)-\beta^3(1-\beta)(4+3\beta)}{32(1+\beta)(2-\beta^2)^2}$	$\pi_R^{2PT} = \lambda^2 \cdot \frac{1}{8(1-\beta^2)^2} \cdot \{\beta(4\beta-3)\Gamma^2 + 2\beta[1+2(1-\beta^2)\Delta]\Gamma - 2\beta(1-\beta^2)\Delta + 1 - 2\beta + 2\beta^3\}$	$\pi_R^{3PT} = \lambda^2 \cdot \frac{1-\beta\cdot\Phi[3\beta\cdot\Phi-2-4\Omega]}{8}$
Total Surplus	$TS^{LP} = \lambda^2 \cdot \frac{96+32\beta-96\beta^2-28\beta^3+23\beta^4+5\beta^5}{32(1+\beta)(2-\beta^2)^2}$	$TS^{2PT} = \lambda^2 \cdot \frac{3-4\beta+2(4-\beta-2\beta^2)\Gamma-(4-3\beta^2)\Gamma^2}{8(1-\beta^2)}$	$TS^{3PT} = \lambda^2 \cdot \frac{3+4\Phi(2-\Phi)+3\Omega^2}{8}$

Note: In Table 1 and Table 2, $\lambda = a - c$.

$(\Delta(\beta), \Gamma(\beta))$ is determined by $\begin{cases} \Delta = \sqrt{1 - \frac{(1-\beta\cdot\Gamma)^2}{1-\beta^2}} \\ \beta^2(2\Gamma - \beta - \beta\cdot\Delta)(1-\beta\cdot\Gamma) = 2(1-\beta^2)\Delta[2-\beta^2-2(1-\beta^2)\Gamma - \beta\cdot\Delta] \end{cases}$

$(\Omega(\beta), \Phi(\beta))$ is determined by $\begin{cases} \Omega = \sqrt{1-(1-\beta\cdot\Phi)^2} \\ \Omega\{2-[2\Phi+\beta(1-2\beta\cdot\Phi)+\beta\cdot\Omega]\} = \beta^2(1-\beta\cdot\Phi)\Phi \end{cases}$

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